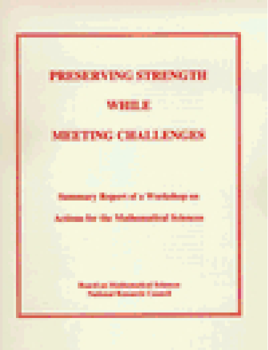

PRESERVING STRENGTH
WHILE
MEETING CHALLENGES

Preserving Strength While Meeting Challenges: Summary Report of a Workshop on Actions for the Mathematical Sciences

Board on Mathmatical Sciences, National Research Council

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THE NATIONAL ACADEMIES
Advisers to the Nation on Science, Engineering, and Medicine

Preserving Strength While Meeting Challenges

Summary Report of a Workshop on Actions for the Mathematical Sciences

Board on Mathematical Sciences
Commission on Physical Sciences, Mathematics, and Applications
National Research Council

National Academy Press
Washington, D.C. 1997

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PREFACE

As part of the global science community, the U.S. mathematical sciences community[1] today faces new challenges in addition to its traditional ones. In common with all of U.S. science and technology at the close of the twentieth century, the mathematical sciences community is now being called upon to respond to different needs even as budgets become more tightly constrained. New accountability criteria have come into play. Academia, business and industry, and government continue to undergo profound changes (described in several of the invited speakers' papers provided here) that affect the community.

In this report, the National Research Council's (NRC's) Board on Mathematical Sciences (BMS) summarizes the results of a workshop held May 17-19, 1996, in Alexandria, Virginia. The workshop, entitled "Actions for the Mathematical Sciences in the Changed Environment," was organized by the BMS with support from the Alfred P. Sloan Foundation, National Science Foundation, and Intel Foundation.

This workshop summary was prepared by the BMS and was reviewed by the NRC in accord with usual procedures. The BMS believes that the views expressed at the workshop should be seen by the mathematical sciences community at large.

AVNER FRIEDMAN, WORKSHOP CHAIR
DIRECTOR, INSTITUTE FOR MATHEMATICS AND ITS APPLICATIONS
UNIVERSITY OF MINNESOTA
CHAIR, BOARD ON MATHEMATICAL SCIENCES
NATIONAL RESEARCH COUNCIL

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[1] "Mathematical sciences" means pure mathematics, applied mathematics, statistics and probability, operations research, and scientific computing (*Educating Mathematical Scientists: Doctoral Study and the Postdoctoral Experience in the United States*, National Academy Press, Washington, D.C., 1992). The mathematical sciences community includes all individuals whose work involves the mathematical sciences, whether it be research, education, application, scientific computation, or administration and sponsorship of such work.

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INTRODUCTION

The mathematical sciences hold a special position in science: They are the intellectual disciplines for the study of abstract structures as well as the language of modeling, analysis, and computation. The mathematical sciences provide the other scientific and engineering fields with the concepts and computational tools so necessary for their advances.

Mathematical sciences research has many recent achievements to which it can point with pride. The proof of Fermat's Last Theorem not only resolved a 350-year-old problem, but also deepened the understanding of important mathematical structures. The mathematical development of wavelets provided new computational tools for important engineering problems, such as data compression and imaging, whose full potential is still being explored and expanded. The recent outpouring of support for the mathematical sciences by leaders in various scientific and engineering disciplines, in response to a (subsequently rescinded) decision to eliminate the graduate mathematics program at the University of Rochester, illustrates how widespread is the recognition of the importance of mathematical sciences research for advances in science and technology. Moreover, it is internationally recognized that U.S. universities are world leaders in graduate-level mathematics education.

A workshop titled "Actions for the Mathematical Sciences in the Changed Environment" was planned as a follow-on to a 1993-1994 project initiated by the National Research Council's (NRC's) Commission on Physical Sciences, Mathematics, and Applications (CPSMA) and reported on in *Dialogues on the Changing Environment for the Physical and Mathematical Sciences* (National Academy Press, Washington, D.C., 1994), and in *Regional Dialogues on the Changing Environment for the Physical and Mathematical Sciences: Report of Two Conferences* (National Academy Press, Washington, D.C., 1995). The purpose of the BMS workshop was to advance those CPSMA dialogues by focusing on and exploring specific issues now facing the mathematical sciences community. The workshop's goal was to lead the mathematical sciences community in productive directions that reflect and respond to the new imperatives facing the scientific community at large and all of society as a result of recent changes in the national and international economic and political arenas.

Nominations for possible workshop participants were obtained from numerous mathematical sciences professional organizations, including the American Mathematical Society, Mathematical Association of America, Society for Industrial and Applied Mathematics, Association for Women in Mathematics, National Association of Mathematicians, Institute of Mathematical Statistics, American Statistical Association, National Council of Teachers of Mathematics, American Mathematical Association of Two-Year Colleges, and Institute for Operations Research and the Management Sciences, as well as from the Mathematical Sciences Education Board. A BMS-designated steering committee, consisting of BMS chair Avner Friedman, BMS Executive Committee member Mary Ellen Bock, CPSMA member (and former BMS chair) Shmuel Winograd, and William Browder of Princeton University, developed a draft agenda and tentative workshop plan and prepared extensive lists of background issues and motivating questions for each of four areas on which workshop break-out groups would focus (see Appendix A). The steering committee also suggested names of people to be invited to lead the break-out groups, and to be invited as workshop speakers.

This workshop summary indicates what issues were discussed as possible actions and reflects the workshop discussions as refined by further consultation with workshop attendees and subsequent NRC review. They are presented as observations of themes raised at the workshop and are for the mathematical sciences community to use as it wishes. *They do not constitute the recommendations of an NRC-appointed committee*.

INVITED WORKSHOP PAPERS

Introductory Comments

Avner Friedman
Chair, Board on Mathematical Sciences, National Research Council

In August 1993 at Chantilly, Virginia, the Commission on Physical Sciences, Mathematics, and Applications of the National Research Council held a workshop entitled "Beginning a Dialogue on the Changing Environment for the Physical and Mathematical Sciences." The workshop was attended by participants from academia, government, and industry. It was followed in June 1994 by two other regional dialogues, held in San Francisco, California, and Vail, Colorado. The following main themes emerged from these three workshops:

1. The need to effect significant changes in both the educational and research missions of the universities;
2. A recognition that the traditional training of scientists may be too narrow from the perspective of society, industry, and, in particular, the scientific community itself; and
3. A desire to move from analysis to action.

Ron Douglas and I participated in the Vail meeting. On the two-hour ride back to the Denver Airport, we discussed the need for the mathematics community to take the lead in a process that will articulate the challenges our community faces within the changed environment, and develop a process needed to position our community so that it can thrive in the new environment.

I subsequently discussed the idea of addressing this need in a workshop; I talked with Shmuel Winograd, Lou Auslander, William Browder, Mary Ellen Bock, and members of the Board on Mathematical Sciences. The Sloan Foundation, the National Science Foundation, and the Intel Foundation agreed to fund this workshop. John Tucker, director of the Board of Mathematical Sciences (BMS), has been instrumental in working out the details.

This workshop is organized as follows.

Today, Friday, from 1:30 p.m. to 5:30 p.m., we shall hear from speakers who will provide information and perspectives on (1) how the public views science and mathematics, (2) how scientists view the role of mathematicians, (3) the challenges for NSF, (4) challenges in the education arena, and (5) areas of new opportunities for the mathematical sciences.

The next phase of the workshop will begin tonight after dinner, with an initial meeting of the four break-out groups as described in the program. Each group will be given a general theme and a set of questions. In the initial meeting the groups may discuss any and all of the themes. Saturday morning each break-out team will make a brief report in a general session. This will set the stage for the third phase, which begins tomorrow at 10:00 a.m., when each break-out group will concentrate on its own theme. Tomorrow at 4:30 p.m. each team will make its final report in a general session. The break-out group leaders, together with John Tucker and myself, will then generate a draft summary document. We shall present this document to all of you Sunday morning, in a general session, for discussion, modifications, and adjustments.

The break-out groups, in addressing their individual themes, are asked to respond in a way that addresses the following general questions:

1. What are the national needs for the mathematical sciences?
2. How can the mathematical sciences community respond to these needs?
3. What resources do we require to do that?

Let me briefly go over the assigned topics.

The first topic deals with communication within and outside the discipline. Several items have been suggested for discussion. The most important are the following:

1. How to establish mechanisms by which the mathematical sciences community can communicate its contributions and values, and
2. How to develop a network of mathematical scientists to maintain regular contact with legislators and the public at large.

Your ideas are needed on how to help mathematics departments maintain and enhance their programs and their standing within the university. You are asked to provide more than general advice. You are asked for mechanisms that will be useful and that can be implemented. One possible suggestion is to set up a regular series of workshops for chairs of departments to address issues concerning the impact of university restructuring and reorganization on mathematics departments: how to anticipate, avoid, and react to such changes.

To increase public appreciation of the contribution that mathematicians make to the nation, our community needs to cultivate leadership. Perhaps a series of tutorial workshops that will deal with science policy and public awareness issues could be run in Washington for young tenured professors.

The second topic is education. Many students receiving a Ph.D. in research universities cannot find jobs for which they were trained. Around the country, graduate programs in mathematics are shrinking and some may close down. The report of an NSF workshop held June 5-6, 1995, *Graduate Education and Postdoctoral Training in the Mathematical and Physical Sciences*, recommended that departments broaden their educational program by introducing more multidisciplinary courses and off-campus experience. If a mathematics department wishes to introduce such a program, how can it be facilitated? This group could, for example, recommend that the professional societies develop a database or a sequence of workshops that feature case studies on how some departments made the transition, what the obstacles were, and where they succeeded.

Going to the third topic, funding for research and education, here are two items you may want to think about:

a. In a recent meeting, mathematical sciences program directors of the Air Force Office of Scientific Research, the Office of Naval Research, and the Army Research Office told the Board on Mathematical Sciences Executive Committee that "more and more money goes to multidisciplinary research," and that "many attractive topics of mathematical content are being 'hijacked' by nonmathematicians" since mathematicians do not pay enough attention to upcoming new programs. How can the mathematical sciences community better take advantage of new funding opportunities?
b. What is the best strategy for apportioning the budget of the NSF Division of Mathematical Sciences? By "best," I mean so that the mathematical sciences community can best

> respond to national needs. I am not suggesting that we resolve this question today. But you may want to consider a recommendation to set up a committee that for a period of, say, 6 months solicits views from all members of the community, and then comes up with strategic guidelines on priorities.

Finally I come to the topic of evaluating performance. The Joint Policy Board of Mathematical Sciences published in 1994 a report titled *Recognition and Reward in the Mathematical Sciences*. The next step is to examine whether departments are doing a good job or can do a better job in evaluating programs in (1) education (K-12, multidisciplinary) and (2) interdisciplinary research. For example, might a national rating be developed that will help mathematics departments?

I do not wish to give the impression that what is needed is an ever increasing number of committees. Our community has new challenges that it must respond to, and your ideas are needed on how to go about it.

Finally, I want to emphasize the importance of this workshop to reach a consensus. It will not serve the mathematical sciences well if we are divided in our recommendations. At this time of stress from so many directions, this workshop can succeed only if the community moves forward in unity.

Expectations and New Opportunities at the Division of Mathematical Sciences

Donald J. Lewis
Director, Division of Mathematical Sciences National Science Foundation

Thank you for the invitation to participate in this workshop, "Actions for the Mathematical Sciences in the Changed Environment." I very much appreciate what the Board on Mathematical Sciences is seeking to achieve by this workshop and commend the Board for its efforts. I will address the issues implied by the title assigned to me by the organizers, but I want to do so in a somewhat broader context.

As you know there is a bipartisan agreement to balance the budget, and at the moment, defense and the entitlement programs appear to be politically untouchable and there is a mood to reduce, not increase, taxes. This leaves only the discretionary programs to Carry the burden of reaching a balanced budget. Both parties in their outreach projections have targeted the National Science Foundation with at most a 3% increase per year, probably the rate of inflation and maybe less. So at the very best, in constant dollars, NSF's funding will be flat—just as it has been for the last 5 to 6 years. This is optimistic, for as you probably know the American Association for the Advancement of Science is predicting a substantial drop in NSF's constant-dollar budget.

NSF Currently provides 52 to 55% of the federal funding for mathematical research and a considerably smaller percentage in the case of other sciences. These percentages could change radically in the next few years. All the Department of Defense agencies are reviewing their basic research portfolios, and it is rumored that there are pressures from the admirals and generals to reduce substantially or eliminate 6,100 budget accounts—the accounts that fund basic research. If this should occur, all the basic sciences will be hit severely, and the proposal pressure on NSF will be enormous in all the sciences.

The foundation is a very conservative organization and if you look back over its 45-year history you will find, with a few exceptions, that the percent of its budget going to any particular discipline has been pretty constant. One exception was a 15% increase in DMS's budget in response to David I (NRC, 1984); others were in response to the Mansfield amendment, and to decisions to build major facilities. As one famous chemist remarked, "God decreed in 1950 the distribution of resources amongst the sciences and no one has dared to go against the decree." So while I see a growing awareness within the foundation that the mathematical sciences are underfunded relative to their importance to science, I am not optimistic that there is currently the will or the capacity within the foundation to double DMS's budget, which would alleviate some of the pain now being experienced, or to triple it, which would put things in a real comfort zone. At least, there will not be such a will or capacity until the scientific community and especially the mathematicians convince the general American public that investments in science and mathematics will improve their lives. The public believes that the National Institutes of Health is working for the improvement of their health, and so NIH thrives even in a bad budget climate. We need to make the case to the corner grocer and the factory worker why their money should be invested in the mathematical sciences.

I do not wish to convey that we cannot make DMS's budget grow; we can and did increase it this year. But the growth will need to be incremental and will come because we have been innovative and are demonstrating careful management of the funds entrusted to us. The mathematical community will need to identify opportunities and help sell them. It probably will need to help identify the more mature areas, and those that have the greatest promise, and be prepared for DMS to do some shifting of funds. As a community, we will need to determine how best to use the limited funds provided to maintain and advance the discipline. This may mean arriving at alternative funding patterns. If we are not prepared to make hard choices, how can we expect others to do so? When the community is willing to make choices and move forward, suggesting areas of opportunity and responding to innovative modes of funding—as, for example, it did for the Group Infrastructure Grants (GIG) program—the foundation does respond with increased funding. The mathematical community must understand the changing environment in which we live, and then be agile in taking advantage of opportunities presented. The purpose of this meeting is to begin the process of taking charge of our destiny.

You should be aware that Congress is now requiring greater accountability of the agencies it funds. The NSF needs to come up with a set of goals and metrics to measure achievement of these goals. Crude counting, whether of students produced, papers published, and so on, will just distort the purpose of the foundation. For that reason it is seeking to have qualitative, rather than quantitative, measurements.

The foundation was founded to promote the progress of science, to advance the national health, prosperity, and welfare of U.S. citizens, and to secure the national defense. The national populace is the foundation's constituency, not the academic research community. Thus it is not sufficient for the NSF to ensure the welfare and progress of academic researchers, confident that their brilliance and hard work will take care of the needs of the nation. It needs to lead the community of researchers and educators in the most beneficial directions for the nation. That being the case, in a qualitative manner we will need to address how our funding has been beneficial to the nation. NSF is funding basic research, and so the time lines for this assessment cannot be one or two years, but since many lines of inquiry will not have a payoff, ever, it will not be convincing to Congress if every investment takes in excess of 50 years before there is any indication that the investment has had a payoff. As Eugene Wigner so ably asserted and demonstrated, mathematics is inordinately effective in its contributions to science—and, as we now know, to management and industry. In the years ahead we mathematicians will need to document this time and time again, and we will need to assist in a faster transfer of knowledge.

As I see it, the mathematics community is going to have to get into the business of telling its story and describing the impact it has had on the general populace, not only to Congress but, to the extent possible, to the person in the street. For far too long, we have been content to communicate individually with a small, select group of colleagues, making little effort to communicate even with the mathematical sciences community as a whole, let alone with our science colleagues. To the educated public we appear to be a small sect, inwardly looking, muttering in an incomprehensible language. If this is the case, how do we demonstrate our contributions to our nation's citizenry?

Our fellow scientists need our help. They need to reduce experimental trials, and they need models to simulate experiments and suggest the most appropriate areas of inquiry. They need the mathematician to help not only in modeling and simulation but also in reformulating their questions. Formulating precise questions is our forte. We have become so inner driven that we have forgotten our ancestry; we forget how much of mathematics was stimulated by physics.

Today every area of science from the physical, to the biological, to the social, and to the managerial is becoming highly mathematized. Researchers in these areas need our help and, in turn, we will discover new areas of challenging mathematics. As I wander about the foundation, I find division directors excited about the idea of mathematicians collaborating with their researchers—excited not just in words, but willing to put up money to back up their words. If more mathematicians were to become active in multidisciplinary research, we could more quickly transfer mathematical discoveries and speed up the time line from mathematical discoveries to contributions to our nation's populace. Let me add that the mathematical expertise for multidisciplinary research should go beyond that of the analysts and computational mathematicians. All the subdisciplines of mathematics have a role to play and need to get involved.

Currently the easiest way to increase the DMS budget is via multidisciplinary research. Many fields of science have progressed to the point that it is easier to get new results by combining ideas from several disciplines, and frequently such results are closer to being useful for the well-being of society. Also, with minimal increases, if funding goes to a multidisciplinary area, then two or more disciplines benefit—thus providing the director with increased brownie points. Areas currently under discussion for possible new funding that would involve mathematics are communications, mining of massive data sets, and machinery for predictability. Each of these should involve several programs within mathematics. We also expect to see growth in materials science, mathematical biology, and computational science.

But the returns to mathematicians from entering into multidisciplinary research are far greater than increased funding and better assessment reports. There are the intellectual opportunities of opening new areas of basic mathematical research. The next half-century could be the golden age of mathematics both as an enabling science and as the queen of intellectual achievements. The National Research Council will soon undertake an in-depth study of the interplay between mathematics and science, looking at how mathematics enables and serves science and, in turn, finds new challenges. Recent interchanges have led to the development of the new field of quantum geometry. There is no one who would not consider it as belonging to core mathematics, and yet it will have deep implications in physics and cosmology. We believe that as mathematicians become more open to multidisciplinary discourse they will not only be good handmaidens but also will find exciting new areas of mathematical research. The opportunities are there. Will we seize them or squander them?

The integration of science and education is a theme we hear of daily within the foundation. It has become the perception of Congress and many citizens that the foundation's funding patterns have compromised the mission of the research universities, causing a bifurcation of their duties and of their staff into two groups—researchers who function as in a research institute, on the other hand, and a lower class that transmits knowledge, often in rote fashion, to the undergraduates. Further, there is the perception that our faculties have failed to convey the excitement and intellectual stimulation of inquiry, and have failed to provide students with hands-on experience in scientific inquiry—skills needed not only by those who become working scientists, but also by other workers and by citizens. This criticism of the foundation is not new. In 1965, after an extensive series of hearings by the House Science and Astronautics Committee, it was reported that serious scrutiny was needed regarding the balance between the foundation's support of teaching and education and its support of research. The foundation and the National Science Board were challenged to take a more proactive leadership role in setting science policy on such matters. The current foundation leadership is seeking to inspire and motivate the research community to address the issue of its role in ensuring that undergraduates learn the benefits and comprehend the method of scientific inquiry. Whether it is accurate or not, the perception that research universities are not functioning properly is undermining university funding by state legislatures and putting many at financial risk. The problem thus extends beyond the foundation and needs to be addressed in order for universities to survive.

The mathematics community has a better record than most of the other scientific disciplines regarding concern for undergraduates. But this record is spotty; not all departments have been involved. Too often our instruction has been too algorithmic, too formalistic, and unchallenging, and has required little intellectual involvement by students. Too often in the past, and even now, we would be hard-pressed to prove that we have taught students to think as mathematicians. Too often we have acquiesced to the use of large lectures and poorly prepared lecturers and temporary faculty. All too often we have been content to be the cash cow for our colleges.

As we know from the University of Rochester experience, failure to look after the undergraduates can undermine the viability of an excellent research group.[1] As you surely know, Rochester's math department was (and is) not the only math department undergoing close scrutiny. When attention was paid to the undergraduates, the departments under scrutiny not only were kept intact but also frequently were given additional funds.

As mathematics faculty we need to be aware of the accounting changes rapidly spreading over the country. With these accounting systems, each unit sits on its own bottom. Tuition income and indirect costs need to balance salaries, supplies, equipment, and, in some cases, libraries and faculty maintenance and heating costs. Other units are free to provide their students with mathematical instruction in any way that meets their needs. Hence it is tempting for engineering colleges and others to build a contingent of adjunct lecturers to provide mathematical instruction, cutting the cost below what a mathematics department might charge under these new accounting procedures. If we do not provide top-quality instruction, we will lose much of our service instruction. Considering the ratio of credit hours earned by in-service teaching compared to that for majors and graduate students, it is clear that service teaching has been an important source of income. Can we afford to lose it?

At present the NSF provides very little funding for mathematics graduate students (about $10 million including fringe indirect costs). This is low compared to the other sciences. But it is nigh impossible to make the case for increased funding of graduate students in mathematics when 15% of the new doctorates are unemployed, and perhaps as many are underemployed, and the time to degree keeps lengthening. In contrast to other disciplines, we have created a culture where academic employment is the only honorable employment. Yet we see time and time again that industry wants, needs, and values mathematicians. If we are concerned with the nation's well-being, don't we need to address how to provide industry with the mathematicians it needs and some of its citizens with the skills to contribute?

A year ago the Directorate of Mathematics and Physical Sciences had a workshop on graduate education and postdoctoral training (NSF, 1996). Mathematicians need to consider the resulting report and respond to it or, alternatively, decide on better principles. I expect MPS, within the month, to fund several innovative graduate math programs as demonstration projects, and more next year, if they are forthcoming from the community.

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[1] See the paper by Stillinger for a brief description of this experience.

One problem the community needs to address is its current dependence on foreign graduate students. There is clear evidence that as the Pacific Rim continues to develop its science investment, many of those educated in the United States will return home. This could deplete our current ranks of top-level young researchers and will certainly lead to overseas economic challenges. Has this dependence really been in the nation's best interests? We once produced excellent researchers who were U.S. citizens. Why now the dependence on foreign students? Have we failed at the undergraduate level? Have we taken the easy way of preferring the more strongly prepared student? Can we defend our actions to the American public in terms they understand?

As mathematicians we envy the way astronomers have developed support from the general public for their research, which we view as being just as esoteric as anything mathematicians do. We hear that they succeed because one-third of the citizens once owned a telescope, even if it did no more than show faint details of the moon's surface. We hear that many college students study introductory astronomy to satisfy distribution requirements and so have had some exposure. We fail to remember that a far larger percentage take calculus, and that when colleges introduced quantitative reasoning requirements, we let other disciplines respond or we offered college algebra. The newspapers run mathematical puzzles each week, which suggests that there are readers who find such things interesting. So, there is at least a latent interest in mathematics in quite a good percentage of the reading public.

Years ago Dick Otter and I offered a course in number theory to freshman humanities students in which the students computed, conjectured, and often made proofs. They experienced what it was to be a mathematician. These students consistently asked for the second semester course, and in the senior poetry publication they made numerous references to number theory. Every time a readable book like *Chaos* (Gleick, 1987) appears, it makes a reasonable climb on the bestseller's list. Have we totally missed opportunities to build a supportive group for mathematics among the general public? Can we learn from the astronomers? Do we need to train and encourage those who can communicate about mathematics to the general public?

I have raised the issues of education, communicating with the general public, especially the educated public, and multidisciplinary research. I have not said much about inner-directed research because I believe we all agree on its importance. I do feel that each of the activities mentioned will flourish only if all the others do. Individually, we probably cannot be involved in all these activities at the same time, and some individuals may participate in only one or two in their lifetime. But if all are necessary for mathematics to flourish, then as a community we must see that each activity thrives, and honor those who contribute to any one of them we cannot have one activity being viewed as more superior.

Funding modes need the community's attention. Currently, DMS is overloaded with proposals; it funds fewer than a third of those submitted, and funds them very inadequately—all this despite the fact that many mathematicians do not even submit proposals because of the conviction that to do so is pointless. The program officers have no time to visit and find out what researchers are doing. As a community we should be looking for alternative, efficiently managed modes of support that would advance our discipline. Would departmental grants serve that goal? They would be a more efficient mode, and they would allow us to consider both the education (graduate and undergraduate) and the research effort of a department as well as expository and informal communications efforts. Your thoughts on this approach could be informative.

I look forward to your discussions and final report, and hope they are directed to maintaining and enhancing the discipline we so love.

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Communicating Mathematics to the Public

Michael Schrage
MIT Media Lab/Sloan School; Columnist, Los Angeles Times

Three statisticians go rabbit hunting. They spot a rabbit. The first statistician shoots—and misses the rabbit's head by 12 inches. So the second statistician takes aim and fires—and misses the rabbit's tail by a foot. The third statistician immediately cries out, "We got 'em!"

Here's another. How about that Sydney Harris cartoon of two besmocked scientists—mathematicians?—looking at a blackboard smeared with a notational riot of Greek letters and equations. In one frame, the two look deathly somber; in the next, they're having a terrific laugh.

These two bits of "math humor" neatly define the pop culture stereotype of mathematics and mathematicians. On the one hand, mathematicians are idiosyncratically precise—but not, alas, inherently connected to the realities of everyday life. On the other, they're breathtakingly obscure and complex—so obscure and complex that only real insiders can get the joke.

Unfair? Unfunny? Humor, relevance, and elegance are all in the mind of the beholder, of course. In the mass marketplace of ideas, however, mathematicians have very little to do with how people perceive the humor, the relevance, or the elegance of mathematics. The public context of mathematics—particularly sophisticated mathematics—is awkward and ill defined. Even worse, much of the public holds what might charitably be described as a "passive-aggressive" attitude toward mathematics. People—especially those liberal arts grads who managed to escape from university without taking a single math class—warily respect what they don't understand even as they quietly resent the impenetrability of the numbers. Mathematics as mathematicians see it is not seen as part of America's culture—pop or otherwise.

This is not a healthy situation. This is a bad situation. In short order, it could become a crisis situation. Already, the culture of mathematics has devolved into subcultures of mathematicians. Within a generation, the mathematics community could splinter into a collection of cults—arrogant, brilliant, self-righteous, and effectively divorced from mainstream issues, empathy, and support. At a time when a major research university declares its desire to jettison its graduate mathematics department, at a time when a math professor *cum* math popularizer writes a *New York Times* op-ed piece asserting that the roots of the suspected Unabomber's madness might be found in the fact that he was a mathematician, there should be no question that even the mathematics community is now asking itself how it should be perceived by America's mainstream media and institutions.

This essay—and this is more an impressionistic essay than hard analysis—is an effort to identify key public perception issues facing the mathematical sciences community and to offer some tentative approaches to dealing with them. This is written from the perspective of someone (emphatically not a mathematician!) who is intimately familiar with how both the mass media and the elite media cover science and technology. Some of the generalizations may seem more rhetorical than empirical, but their purpose here is to evoke—and provoke—useful discussion on how mathematics is perceived by its relevant publics. The accent should be on the word *useful*. The goal is to identify how the mathematics community can influence both its public policy and public perception agendas.

While it is undeniably true that many of the math community's problems are self-inflicted, it is equally true that much of the difficulty lies in the nature of mathematics. Mathematics is hard. People are intimidated by the complexities, thought processes, rigor, and unforgiving precision that mathematics demands. For the overwhelming majority, math is not a subject that lets you be an autodidact.

To make matters even worse, mathematics remains one of the most poorly taught subjects in both K-12 and university education. Indeed, the nature of mathematics is such that one can have superb teachers for five or six consecutive years—but that one poor teaching experience can effectively undermine both the interest and the ability to further pursue the subject. Efforts to reform the math curricula have invariably led to disappointment. The practice in many schools of singling out the mathematically gifted and putting them in accelerated classes or special programs further underscores that math is the province of a quantitative elite rather than a discipline that should be relevant and accessible to the mainstream.

Even those who perform well in mathematics do not necessarily like the subject; and people who perform at average or below-average levels tend to dislike and avoid it. Indeed, it is shocking how few public high schools and universities have instituted anything beyond minimal competency requirements in mathematics for their nonscience students to graduate. While virtually everyone in the educational establishment acknowledges the "importance" of mathematics, there remains absolutely no agreement about how that importance should be mapped onto a curriculum. Even if there were consensus, do the schools even have the teachers who could effectively communicate that?

That said, the ideal of a "mathematically literate" public becomes completely ridiculous. The pursuit of a mathematically literate population is doomed to be a dead end and a waste of time. Yes, there is no consensus as to what constitutes mathematics literacy. But, more importantly, even if some sort of realistic criteria for math literacy were defined and the resources were magically there, just how long would it take to achieve it? Just how much would today's math-indifferent population care about their degree of math literacy? Indeed, even if the population were not indifferent, just how long would it take to acquire the skills and knowledge to become literate? The harsh reality is that the educational foundation for mathematical literacy is just not there. So the challenge becomes what to do when appropriate literacy is neither possible nor practical.

To further complicate the challenge, the mathematics community has grievously wounded itself by alienating the very communities that should be its natural allies. One Stanford engineering professor recalls, with incredulity and annoyance, taking a grad math class at the California Institute of Technology because his math friends said that the professor was tops in the field. The engineer did poorly in the class because his proofs weren't deemed "elegant" enough by the math professor. To this day, he speaks of the episode with a mix of bitterness and annoyance.

Indeed, many science and technology students who reside two full standard deviations above the popular norm in math skills and knowledge speak resentfully of their math education in university. They feel as if they are treated as stupid general contractors and handymen by the mathematics architects—as people who are competent enough to build, but not gifted enough to appreciate, the "true" value of mathematics.

Consequently, even people who inherently appreciate the importance of mathematics as a discipline are less than enamored of the mathematics community as its guardian—they learned in spite of, rather than because of, the math department. Indeed, the past 20 years have seen engineering departments and computer science departments gradually take over the quantitative training of their graduates with only minimal participation by mathematicians. Thus, math departments are being dis-intermediated by the very constituencies that should be looking to them for guidance and insight into the future of computation and analysis.

The essential point is that mathematics may be an extraordinarily powerful and important intellectual discipline, tool, and adventure, but the mathematical community is not seen as particularly powerful or important in helping others acquire or appreciate math as a discipline, tool, and adventure. G.H. Hardy's *A Mathematician's Apology* is a sobering reminder of a key paradox—too often, mathematicians with enormous influence outside their field choose to dismiss their influence in favor of a focus on their own work. They treat "relevance" more as a disease than a virtue.

All this is written to create the context for what follows: mathematics doesn't just have a public relations and/or communications problem—it has real problems. Those real problems are faithfully reflected in its poor public relations and dysfunctional communications. Any meaningful discussion about how to improve the quality of communications and dialogue between the mathematics community and the rest of the world must begin there.

With the exception of stories like the four-color problem or Fermat's last theorem, mathematics generally receives practically no useful coverage in either the scientific or the popular press. Of course, stories like Fermat's last theorem are more about the artifact of the problem and the unique tale underlying it than any intrinsic appreciation of the mathematics. Similarly, the four-color problem is sufficiently easy to explain (who hasn't seen a map?) and controversial (should computers be allowed as part of rigorous proofs?) that crafting an accessible story poses a worthy challenge rather than an arduous task.

Let's take as a given that the community of journalists is woefully ignorant in mathematics. Intriguingly, that really shouldn't be a barrier in media coverage. The media are filled with people who are writing about topics they know very little about. Content knowledge rarely determines who gets to cover what. This is something that many specialists simply do not understand about the media culture. On the contrary, they point to media ignorance as the very reason why the media (and, indeed, the public at large) should be held in such contempt.

At this point, I'm compelled to remember something that Ben Bradlee, my editor at the *Washington Post*, once said: "We don't print the truth; we print what people say." The reality of media coverage is that most journalists—whether brilliantly experienced or breathtakingly naive essentially write (or tape) what people say. They count on their sources to generate the story and identify what's important. The best stories in the popular and elite press are almost always shaped, determined, and influenced by the quality of their sources.

The quality of the sources—their ability to quickly respond to media inquiries; to frame responses in ways that make the story easier to produce; to encourage the journalist to expand or narrow the story in a way that reinforces what the journalist is really trying to do, and that respects the limitations of the medium and the journalist rather than treating them as excuses for dismissal—is the single most important criterion for determining the quality of media coverage. Sources shape stories more than stories shape sources. Period.

Much of the inherent problem with mathematics coverage is that, more often than not, it's difficult to pick the right metaphor that enables a math story to be both accessible and compelling. Physics, chemistry, biology, and technology all lend themselves better to metaphor and analogy than does mathematics. Mathematics does not intrinsically lend itself to narrative. Mathematicians do not "construct" stories or narratives in the ways that, say, sociobiologists, molecular biologists, or computer scientists do.

To see what happens when media culture collides with math culture, consider the very words mathematicians use when describing their discipline. Mathematicians often refer to mathematics as a kind of a language with its own subjects, verbs, adjectives, and nouns. Indeed, mathematics is both literally and figuratively a dazzling array of many languages, dialects, and creoles.

Now, ask yourself: How often are languages covered in the popular and elite press? How often are new French phrases quoted or Japanese kanji cited or Chinese ideograms explained in, say, a month's worth of *Wall Street Journals* or *Fortune* magazines? When are translations of Russian graffiti and New Right German neologisms a part of videos done by CNN or ABC?

The answer, of course, is almost never. Yes, the media cover France, Japan, Germany, and China as *countries* and *cultures* but rarely do they focus on the languages. Mathematics is covered not as a community or as a culture but as a *language* and, as a result, it falls into the journalistic mindset of something that is seldom written about by the mainstream press. (However, a notable exception to this is how jargon seeps into everyday language —it's easy to pitch a story about how technical slang from computers enters colloquial speech, or how medical terminology from a hit show like "ER" becomes a linguistic meme. If certain math terms began to seep into the pop vocabulary, you can be sure there would be a *New York Times* story and a tongue-in-cheek CNN feature within 6 months.)

Similarly, another term that mathematicians use to describe their favorite work is "elegant." They describe the intrinsic "beauty" of their math; they describe a genuine, heartfelt aesthetic. So what's the metaphor here? It is the metaphor of "high art"—an art so rarefied and obscure that only the most refined aesthetic palates might appreciate it. Now, how often do the mainstream press and the elite media cover the avant-garde canvases and sculptures of little-known European artists? Yes, art exhibits in the mainstream museums and galleries are covered regularly by the mainstream media (although not by publications like *USA Today* or any of the major television networks). But truly "breakthrough" art in the smallish galleries and museums remains the province of—surprise—obscure journals and "fanzines." Indeed, these publications have circulations comparable to peer-reviewed math journals.

In reality, mass media coverage of the sophisticated arts occurs about as frequently as coverage of sophisticated mathematics. The mainstream media avoid obscurantist art (but not, necessarily, obscurantist artists) as rigorously as they avoid obscurantist mathematics (but not, necessarily, obscurantist mathematicians). Mainstream media (even elite media) prefer to cover mainstream—and accessible—art, just as they would prefer to cover mainstream mathematics. What, in the 1990s, qualifies as mainstream mathematics? Stories about statistics and probability; encryption; new algorithms that make computers calculate faster or compress imagery better—that is to say, mathematics that can in some way prove relevant to the mainstream.

The point of these examples is very simple: the very metaphors that mathematicians use to describe their field are metaphors that the media typically ghettoize in their coverage. They are metaphors that limit rather than metaphors that expand. (To be fair about this, it would be difficult for most newspapers to cover math well even if a gifted journalist wanted to get a provocative equation on the front page: at the *Washington Post, New York Times, Wall Street*

Journal, and *Los Angeles Times*, a reporter would have to call either the art department or the computer typesetting people to make sure the fight symbols were laid out in the right order on the page. Getting math notation into a newspaper is not as straightforward as writing a sentence with italics or boldface. There are genuine technical barriers to serious math coverage in the mainstream media.)

Consequently, if the mathematics community wants to dramatically improve its media image, it has to be prepared to invest its intellectual resources in coming up with metaphors that the media can easily grasp. Note that metaphors are not the same as sound bites. Obviously, it's easier to come up with metaphors for statistics and probability than, perhaps, for real analysis.

Then again, the role mathematicians are playing in importing topological analysis to protein folding and molecular biology offers a magnificent example of how once-"pure" math can offer rich insights into the fundamental processes of nature. Is the real story here how mathematics illuminates nature or how nature gives insight into mathematics?

The knee-jerk and gratuitous suggestion is that the math community will find it far easier to peddle stories about the growing "relevance" of mathematics to core disciplines that people care about, i.e., medicine, nature, ecology, computers, software, encryption/privacy, gambling, economics, finance, and so on. To be sure, the mathematics community has done a miserable job of doing even that.

However, a more challenging context for mathematicians to explore would be to market not "relevance," but "perspective." What kind of useful and valuable perspectives does the mathematical mindset offer? What is the relationship between the culture of mathematics and mathematicians and the kinds of perspectives they bring to bear on problems in other disciplines? How do other disciplines—medicine, economics, meteorology, and so on—beg, borrow, and steal from innovations in mathematical thought?

Similarly, the math community should take the risk of being covered as both culture and community. In the same way that the "cold fusion" controversy illustrated many useful and intriguing insights into the rivalries between physicists and chemists, perhaps it might also be useful to let people see the schisms between, say, "pure" mathematicians and industrial mathematicians, and mathematicians and statisticians. In other words, let the media use the culture of math as a lens to look at math as a discipline.

Indeed, the role that mathematicians played a while back in blocking Samuel Huntington's elevation to the NAS was, in retrospect, an opportunity missed in helping establish in the public mind what values mathematicians hold as scientists.

With these points in mind, here are specific suggestions on options the mathematics community might consider to help influence both perceptions and policy in the media:

1. Give up the notion of mathematical literacy in favor of mathematical awareness. What trends and ideas in mathematics should taxpayers and mainstream Americans be aware of over the next 2 years? Stress the value of the perspective mathematics brings to various disciplines rather than the substance of math itself.
2. Identify no more than 20 media outlets, that is, highly targeted sections of broader media outlets—the Science section of the *New York Times*; the Finance section of *Business Week*; "Science & Technology Week" on CNN; etc.—that reach what the community deems to be a desirable audience. Analyze which stories produced over the last 6 months could have been

improved by a quote, contribution, or guidance from a mathematician. The point is to *leverage existing coverage* rather than try to create a new genre or set of stories about mathematics itself.

3. Develop a working group of 10 to 12 people who are familiar with trends in math itself and who will try to express these innovations in metaphors and analogies that are communicable to nonmathematicians. Every gathering of mathematicians should have a working session where participants are funded to come up with a presentation or concept that is designed for a lay audience.
4. Make sure that people can articulate the history and the story behind the idea. It is often easier to explain an idea in the context of its origins than to explain the idea itself. History is inherently narrative.
5. Create one-day workshops for members of the media (and relevant federal and state staff) that invite the best mathematicians around to give accessible tutorials on major math themes. The goal should be not to produce faux literacy but rather to promote a conceptual appreciation of why that particular aspect of math warrants further study. The analogy is Leonard Bernstein teaching people how to listen to Beethoven. People don't have to be able to read music to better listen to it.
6. Parasitize other public relations initiatives by the science and technology community. When engineers or physicists are putting on a press briefing or a conference that attracts media coverage, make sure that mathematicians are there to put their spin on how mathematics is influencing advances in those fields.
7. Make a concerted effort to infiltrate the business press. Between Wall Street quants, economists' simulations, and computation, the quantitative content of the business world is rapidly increasing. Mathematicians should be commenting on how the future of math will shape the future of value creation. In other words, how can math—like microprocessors—be seen as a "mission-critical" part of the information economy?
8. Identify the rivalries and cultural conflicts in the math community and "market" them to the media as signs of an institution undergoing introspection and renewal. *Science* magazine "News and Comment" stories like this invariably get picked up by the mainstream media. The challenge: How can these conflicts be portrayed in ways that give the mainstream greater insights into how mathematicians define their field?

There is no doubt that mathematics remains a vibrant and evocative discipline whose intellectual influence is expanding in spite of the limitations of the community that defines it. It is time that the mathematics community recognized—and acted upon the recognition—that it has a moral obligation to make it easier for people to appreciate what mathematics can be. Knowledge and understanding are too much to ask; appreciation and awareness are not.

A View from Capitol Hill

David Goldston
Legislative Director for Representative Sherwood Boehlert

I am going to talk mostly about the status of science funding and some reactions to it, which are current discussion items in various quarters. This is based on what I have heard at scientific meetings over the years both as a legislative director and when I was on the science committee's staff. I will indicate concerns I hear from scientists and mathematicians, where I think they are well placed and where I think they may be less well placed, and what should be done as a result.

Funding for mathematics and science, especially basic research funding, has really done remarkably well thus far in this budget environment. Last year, despite concerns about the long term, the National Science Foundation held its own fairly well. The Department of Energy programs, which had been much more under attack over the years, ended up coming out fine in terms of basic research. The National Institutes of Health had (as was indicated before) a healthy increase. So everything worked out pretty well despite a lot of trepidation in the community that science was being targeted. This does not necessarily mean times are easy. Proposal pressures are unbelievable, and young scientists have terrible problems getting started. But in terms of macro figures, science and mathematics have done much better than any political science review of the factors influencing the budget would lead one to predict. The questions are, Why has that been the case, and how does that relate to the future?

There are three reasons that scientists have done well so far, none of which should give much cause for comfort. The first one is that, despite some of the paranoia one hears at scientific conferences, science and mathematics are not on anybody's hit list. They are not targeted (although technology programs are a different story). But the people on the Hill who know and think about basic science and mathematics research problems are extraordinarily supportive, and nobody else on the Hill knows those issues exist. That is really the way it has always been; it is not a new phenomenon. That is the way these problems are addressed by Congress, and they are viewed relatively as being as close to sacrosanct as anything can be on Capitol Hill. In a meeting this morning with a senior appropriations staffer who oversees NSF funding, we were reviewing all the problems coming up this year with the Housing and Urban Development, the Department of Veterans Affairs, the Environmental Protection Agency, and similar things. He just mentioned in passing, "And, of course, no one wants us to touch the foundation." So NSF is in some ways in the eye of the hurricane within appropriations. There has been no targeting of basic science, and that is good.

Another, related reason is that science and technology funding have had specific champions on the Hill. Often such people are senior members of Congress, and that has been very helpful because, as I said, there is only a small community of congressional people who really watch these issues, who know that they exist, and who care about them. And those people support science.

The third reason is that, even though it does not feel like it today, in all the budget-cutting scenarios these are, in fact, the "easy years." All of the proposals are for 6 years of budget cutting, and they are all back-loaded. The Administration's plan is slightly more back-loaded than the congressional proposals, but all the cuts are—not surprisingly—in the future. If we stay on this track, as it now seems we actually will, every year is going to get tighter. What that adjective "easy" means is that budget cutters have only have had to go after things that have long been targets of opportunity or targets of ideological criticism.

These three factors explain why things have worked out so well thus far for basic science. However, the one common trait they share is that none gives much hope for the situation continuing. The easy years are obviously followed by hard years. You do not even get 7 fat years; you get maybe 21/2 fat years and then 5 lean years. The champions of science are going to be retiring, and so Congress's historical memory in the area of science is evaporating. Also, not being targeted is great, but it also means that people on the Hill do not necessarily know a lot about what is happening in science when budgetary constraints get tougher, and when having political support depends on being known and appreciated.

Consequently, we have a window of opportunity to build more support for basic science programs. Even more than that, the primary issue is not building more support for these programs, but figuring out how people in your community are going to survive in this different environment. That is an internal question rather than an external question. People like to focus less on the internal question because it is much easier to focus on how to do a lobbying campaign using the springboard of mathematics society meetings. But lobbying is not going to be the answer, because the hard numbers do not allow any positive scenarios for the future. The good news that we have had thus far does not mean that such things are going to continue. What we have is not a cause for complacency, but a small window of time in which to address both internal and external issues.

In thinking about what to do, there are three fallacies that need to be avoided. One I call the internalist fallacy. This is the belief that everything that happens to science and mathematics is about science and mathematics. This kind of thinking assumes that math and science exist separately from the world at large, and that if funding for science and mathematics is being cut, then that has something to do with scientists and mathematicians and their attitudes. That sort of thinking is ridiculous. There is a comprehensive budget situation, and there is an overall political situation. Science and mathematics for most people in Congress are not even background noise. If you disagree with long-term congressional budget scenarios, then write your congressman or Congresswoman and say that you do not want the budget balanced in 6 years. But do not write to them saying, "I want more mathematics and science funding." The Administration and the Congress are committed to balancing the budget in 6 years, although no rationale has been given for why it must be done or done in that time. That 6-year balancing is now the political context in which all debate takes place. It is important to avoid viewing things only in terms of your piece of the world, because that is not where the primary forces are that are generating the problem.

The second fallacy is what I call the Vannevar Bush fallacy, the notion that all we really need now is another report like *Science: The Endless Frontier*. I worked with the Council on Competitiveness while it was trying to write such a report, and it is a very appealing thing to do. But this approach reflects a total misunderstanding of the role that the *Endless Frontier* report actually played, what it meant, and how that time differs from the present. This is more than just a nice historical point because it does affect the way people think. There is the sense that if we issue the right report, then everything will be fine again because that is the way it worked in 1945. Following that 1945 report we were happy for 40 years. Unfortunately, that is not true. First, what the *Endless Frontier* report said is that what worked well during the war, worked well, so let us keep doing it. That is very different from trying to come up with new solutions.

Second, except for its fundamental, essential point that the government should have an important role in science and mathematics funding and support, none of the report's actual recommendations—ones that Bush thought were important and cared about—have ever been put into place! There is no department of science and technology. There is no system of math and science governance that is insulated from political forces. None of that has ever happened. The third point is that the present system did not come into place the day after the *Endless Frontier* was released. There were about 4 years of very nasty debate before the National Science Foundation was established in 1950. NIH got built up on a separate track, and so did DOD (because of Cold War concerns). Hence, the world did not then and does not now spring up out of a box; you do not get a little kit and put it together to produce structure. Waiting for a magical report that is supposed to solve our problems is silly. Yet I hear hints of that in the background of a lot of discussions. It does not mean that writing those kinds of reports is a waste of time or that the *Endless Frontier* report did not have significance. But relying on report writing as a solution reflects misunderstanding about how things happen and puts us on an impossible path away from *acting* on the real problems.

The third fallacy concerns communication. These fields do not have funding trouble now because of a communications problem. By this, I do not mean that people should not engage in communicating and do some of the things that Michael Schrage discussed. There should be more of that just so that the public can better understand science and mathematics. Better communication and understanding are needed for all sorts of idealistic reasons, without regard to funding issues. And certainly, *not* communicating will make matters worse, because budgets are going to get tighter and tighter. When the squeeze finally comes, you do not want people on the Hill who have never been paying attention to you to say, "I have never heard of those guys; how important can they be?" However, the problem is not a communications problem; it is a budget problem; it is a problem of numbers. When people develop communication strategies, those strategies usually have the effect of saying, "We are important and we are interested in the long-term health of the nation, not just in helping ourselves; we are not just an interest group, please give us (fill in the amount), and sign here." The odd thing about this is that very few groups come to the Hill and say, "We are just concerned about our own short-term interests, so please increase our budget." The arguments that scientists and mathematicians have to make—and they *do* have to make them—do not distinguish them from other bidders for public support. Teachers make the same argument; people working in the welfare system make the same argument; defense contractors make the same argument. This is because it is the only argument for public support. It is not an argument unique to science and mathematics. Now, these arguments need to be made. However, the idea that what has been lacking is a communications strategy, and that if we just talk more everything will be OK (1) leads to the wrong kind of talk and (2) is going to lead to woeful disappointment because that is not really or solely the issue.

The hard part, of course, is deciding what to do. This is where I suddenly like working in Congress, and I can say, "Well, you do not want us to solve your problems, so go do it." To a large degree, that is my message. I am going to offer some solutions, but this situation of decreasing funding is not going to go away—1968 is not coming back, and neither is 1980. People in your scientific community who know the system well, who live in it day to day, are going to have to come up with reforms that you think work for you, because if someone else comes in with reforms for you, believe me that will not be pleasant.

As part of the communications strategy, one of the initiatives has to involve saying clearly, "Here is what we are doing to change things ourselves, on our campuses, in industries, and in our departments." Once you have said that, then others may be more willing to listen to you say, "and here is why we are still important to the public at large." But if your message is just "this worked really well in 1960, and if you give us enough money it will work really well now," the answer will be that it is not 1960. I wish it were, but it is not.

So, what kinds of things ought now to be done? The answer, which is perfect for this group, is to focus more on education—not to the exclusion of research, but with the intention of changing the culture through students. Do you want to know how? You focus most—and in the case of mathematics, it has to be a *lot*—of the attention on students who are not going to be math majors. Consider all of those people who crowded through introductory calculus classes. Guess what? They graduated—and went on to work in a congressman's office, telling him how to vote on science budgeting. Most congressional staffers, especially those handling issues that are not the top priority for the member, are in their early- to mid-twenties. Maybe new distribution requirements, or at least better advanced placement courses, are part of the answer, but in all seriousness, that is your best entree. Focusing on better educating the non-mathematics majors works better than an article in the *New York Times*, and even works better than a TV program.

If you want to get people interested in your discipline and have them understand why it is important, you have them right there—they are in your classrooms every single day. If you want to change the culture, that is who you should turn to. To press the point, there must be a focus on nonmajors. If all a 25-year-old knows about research is that it has kept her from ever seeing her professor, then good luck getting to meet that person's congressman or ever getting any kind of money. And that is not an unreasonable attitude for a lot of students to have today. If you do only one thing, emphasize teaching, which will have lots of ancillary benefits that are more important than lobbying. This is the one thing that the academic community has to offer that other constituencies do not have, both in terms of selling yourselves and the very important education component for undergraduate and graduate students. You already have direct access to all of the people you are trying to influence—access that so far, social workers, defense contractors, and none of the other lobby leaders have.

One message that I would emphasize, and that is extremely important, relates again to this internal universe of "you." When members of Congress look at science and mathematics, they look at it in terms of their overall attitude toward academia, not just in terms of research. If the academic system seems to be broken in terms of tuition, or in terms of indirect costs, or political correctness, or whatever, that will not be the sole determinant, but it will have an impact on all of you. Members of Congress do not ask "How much do we give to mathematics?" At the appropriations level, they do not get below (or at least we try to keep them from getting below) the funding line for research for NSF, called "Research and Related Activities." Members do not go to the directorate level unless there either is some special problem or some special need they are trying to address. At the authorization level for the science committee, where those individual accounts do have to be considered, we generally follow the Administration. If we lower the macro level, we lower everything proportionally; we generally do not try to second-guess field-by-field decisions.

It is important to recognize these perspectives and to play from your source of strength: you have access to students. For both the students' sakes and for your sake, that strength should not be frittered away.

Physical Scientists Are from Mars, Mathematicians Are from Venus; How on Earth Can We Communicate?

Frank H. Stillinger
Bell Laboratories (a subsidiary of Lucent Technologies, Inc.)

BACKGROUND

The growth of technology, basic science, and mathematics has been intimately intertwined throughout the history of human civilization. The relationships involved have traditionally offered mutual support and stimulation. But as each of these three broad areas of activity has grown in diversity and complexity, unintended barriers to communication and cross-fertilization have occasionally arisen. This essay examines a few aspects of those apparent barriers between the physical sciences and mathematics communities and suggests some remedies.

My personal view of the present situation is doubtless subjective, but it is certainly optimistic. In particular it has been colored by formal training as a theoretical chemist (University of Rochester undergraduate, Yale graduate student), and by nearly four decades of basic research activity in theoretical chemistry, condensed matter physics, and materials science. It also reflects direct involvement in two recent National Research Council reports that examined the interfaces between mathematics and the physical sciences:

- *Mathematical Research in Materials Science: Opportunities and Perspectives* (Board on Mathematical Sciences, Avner Friedman, Chair, National Academy Press, Washington, D.C., 1993); and
- *Mathematical Challenges from Theoretical/Computational Chemistry* (Board on Mathematical Sciences, Frank H. Stillinger, Chair, National Academy Press, Washington, DC, 1995).

In addition, I have also found the following to be personally influential:

- *Recognition and Rewards in the Mathematical Sciences* (Joint Policy Board for Mathematics, Calvin C. Moore, Chair, American Mathematical Society, Providence, R.I., 1994); and
- *SIAM Report on Mathematics in Industry* (Paul W. Davis, Chair, Society for Industrial and Applied Mathematics, Philadelphia, 1995).

Indeed many of the remarks below reiterate points featured in these documents.

OBSERVATIONS

The traditional division of university faculty into separate academic departments has been an educational necessity. But it has had the effect of creating separate cultures, each with its own dialect, behavioral norms, and professional valuation and reward standards. These distinguishing characteristics in turn have been reinforced by the professional societies that serve the respective disciplines. And in some degree the editorial policies of professional journals have acted similarly.

Language differences between disciplines have become a serious problem. Each group has generated its own technical vocabulary (i.e., jargon), and in many instances a given word or phrase carries different meanings in different disciplines. This situation is troublesome enough between, say, materials science and chemistry, or between condensed matter physics and molecular biology. But many researchers report that the situation is particularly acute between these fields and mathematics, especially since the last seems to celebrate the style of expression that has aptly been called "abstract minimalism."

Deeper, methodological differences also should be recognized, beyond mere vocabulary and mode of expression. The physical and biological sciences often concentrate on inductive processes, while mathematics is primarily deductive, and this distinction can cloud opportunities for fruitful communication. By way of extreme illustration, mathematics appears to have no analogs to the purely reconstructive fields of paleontology and archeology that operate on a preponderance of physical evidence but are not amenable to direct demonstration (proof) or refutation (disproof). Put another way, the standards of "proof" can often be very different in the physical and biological sciences as compared to mathematics.

To the extent that these features have created a sense of isolation of mathematics from the rest of the scientific intellectual community, we all suffer. One unfortunate scenario that can result has recently unfolded at my undergraduate institution, the University of Rochester: the administration decided that the mathematics doctoral program could be scuttled as a cost-saving measure, citing as one reason the disconnection between the mathematics department and the rest of the university. Luckily the original draconian proposal seems to have been softened by the effect of widespread outrage from the external mathematical and scientific communities, but it is unfortunate that such a display of solidarity required the presence of a threat.

We need each other. The physical (and biological) sciences would be a dreadful morass of confusing empiricism and blind experimentation if it weren't for well-posed mathematical models to organize data and to generate predictions of novel phenomena. Mathematics would likely drift into aimless pedantry without the real-world focus and drive imposed by science and technology. In the best of circumstances mathematics rises constructively to the challenge of rigorously characterizing concepts and models, and in that process enforces precision of thought among physical scientists who often slide into lazy habits of fuzzy thinking.

We ought to be able to invent relatively painless remedies to reduce the barriers to cross-discipline communication and collaboration. Furthermore, these remedies should not, and indeed need not, erode the distinctive character of mathematics as an estimable profession. The goal instead is to improve "diplomatic relations" between mathematics and the other sciences in order to stimulate research at the interfaces with inevitable benefits for society.

RECOMMENDATIONS

1. The evaluation process for promotion and tenure in university mathematics departments should be revised, where needed, to provide a fair assessment of collaborative and/or interdisciplinary research. Furthermore, students and young faculty should be encouraged to seek research opportunities in these areas, rather than discouraged from pursuing such prospects.
2. Dual mentoring (with both mathematics and physical-science advisors) for Ph.D. programs should be permitted when the research topic lends itself to this strategy.
3. Establish intra-university faculty exchange programs between mathematics and other departments where prospects for significant interdisciplinary research are perceived. This can take the form of joint seminars, or of internal sabbaticals.
4. Require more thorough mathematical training for undergraduates in the physical and biological sciences, and in engineering, with emphasis on nontrivial applications. This might include senior seminars on appropriate topics in mathematical physics, mathematical chemistry, mathematical biology, and so forth.
5. Establish regular contacts between the professional societies representing mathematics and those serving the physical sciences, specifically to explore the prospects for enhancing cross-disciplinary research. An expected outcome would be creation of special symposia and workshops devoted to areas of present and likely future cross-disciplinary activity.
6. Create one or more awards for mathematicians who have produced original and influential interdisciplinary research. Seeking industrial support for such an award might be reasonable and appropriate.
7. Editors of mathematics journals should be encouraged to solicit research articles at least occasionally written in a style more accessible to "outsiders." Also, well-written review articles on broad interdisciplinary subjects need to be commissioned, particularly if they concern prospects for new research.
8. Publicize recent significant mathematics advances to the physical and biological sciences communities, and invite reciprocal action from these communities.
9. Through popular articles, mathematicians need to explain what kinds of problems, concepts, and activities engage their interest and motivate them, and they should clarify how these attributes may distinguish them from other technical professionals.
10. Prepare and distribute a guidebook for mathematics students and for prospective employers in government and industry exploring the full range of valuable application areas for mathematics, including specific case histories.
11. To ameliorate language problems between the disciplines, establish a two-way online glossary of technical terms and acronyms between mathematics, on the one hand, and the physical sciences, on the other hand. It is most appropriate that this be implemented by joint action of the professional societies [see item 5 above].
12. Take appropriately assertive action to ensure that final recommendations from this workshop are publicized and implemented. This should include presentations at professional society meetings, as well as mailings to department chairs, deans, government agency personnel, and selected congressional staffers.

The Growing Impact of Mathematics in Molecular Biology

Michael S. Waterman
Department of Mathematics, Department of Biological Sciences, and Department of Computer Science University of Southern California

INTRODUCTION

The second half of the twentieth century has been a time of great scientific progress in biology, starting with the discovery of the double helix by Watson and Crick in 1953. Dramatic breakthroughs are reported in newspapers and popular magazines as well as in the scientific literature. Despite the heady times biology has been experiencing, these are early days: the next century is likely to be a golden age of biology, when scientists will build on the fundamental progress made recently. In this short paper I will sketch some of the areas in molecular biology where mathematics and theoretical computer science have had genuine application. (By *genuine I* mean that the application has actually mattered to scientific progress.) Fortunately, the Board on Mathematical Sciences recently sponsored a project that resulted in a book published by the National Academy Press, *Calculating the Secrets of Life* (NRC, 1995), that treats these topics, the subtitle being *Applications of the Mathematical Sciences in Molecular Biology*. Therefore anyone wishing more complete treatments can start with that book (which is not too expensive either!). In addition, I have written a textbook, *Introduction to Computational Biology* (Waterman, 1995), at the beginning graduate level, which allows a person to systematically learn about a subset of the topics in substantially more detail.

This paper is organized as follows. The Human Genome Project (HGP) is described. This project has attracted many new workers to biology. Then short descriptions are given of several areas where mathematical sciences have had application: genetic mapping, physical mapping, DNA sequencing, DNA and protein sequence comparisons, and evolutionary biology. Other very important topics are not covered due to time, space, and the limitations of the author; they include determination of protein structure, and the topology and geometry of DNA and protein structure.

THE HUMAN GENOME PROJECT

The genome of an organism is all of the DNA in the chromosomes of the organism. The genome contains the DNA of the genes and much DNA that is not genes. The human genome of 3×10^9 letters of DNA uses less than 1 percent of that DNA for its genes. The use—if any—of much of the remainder is unknown. As the DNA is a blueprint encoding an organism, knowing the genetic text is very valuable in the larger context of learning the biology of an organism. DNA is "written" in an alphabet of four letters, *A, T, G*, and *C*, which are symbols for the nucleotides or bases in the linear chain that makes up one strand of the macromolecule. There is

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another complementary strand, where *A* base-pairs with *T*, and *G* base-pairs with *C*. The strands have a chemically defined direction, so that the strands of the double helix are complementary and have opposite polarities. This is the key to molecular genetics; the double-stranded DNA separates into single strands, and each strand is then used to template a new double-stranded molecule. Protein molecules are themselves linear macromolecules that are written in the alphabet of 20 amino acids. Genes then encode proteins using the genetic code, a mapping from triplets of bases to the 20 amino acids (including three triplets that spell "stop").

In 1985 scientists began to discuss sequencing (reading the DNA) of the entire human genome. This was a huge leap from then-current technology. Only about 7 million letters were in the international databases; even if that was only half the then-sequenced DNA, it was a small fraction of the 3 billion letters of the human genome. And sequence length was daunting: a few sequences in the 50,000-letter range had been determined, but they were regarded as extraordinary feats. Reading a 15,000-letter sequence was more within the range of mortals. Even if chromosomes could be isolated, the 3×10^9 length would be reduced by only one order of magnitude. The first discussions were about creating a mega-project, where biology would be done in a factory. By the time the project had its official start in 1990, there was a big-science flavor to the project but the work was very distributed, involving many laboratories. The U.S. agencies funding the HGP are the National Institutes of Health and the Department of Energy; the project was to take 15 years and cost $\$3 \times 10^9$, a dollar per base. The HGP goals include the efficient and cost-effective development of genetic and physical maps of the human genome, the determination of the complete nucleotide sequence of the human genome, and the development of technology to achieve these goals. In addition, genetic and physical map and sequence information is to be collected for several model organisms such as mouse and yeast.

It was not evident to many that these goals were possible in the proposed time frame. Fortunately, progress in genome research has exceeded the initial guidelines, and a complete human genome sequence is hoped for by 2003. (See the recent review by Guyer and Collins (1995).) Some of the mathematical aspects of this project and of molecular biology are discussed in succeeding sections. GenBank is the U.S. public DNA database. Figure 1 shows GenBank growth since 1984; it is an indication of the explosion of knowledge in the biological sciences.

GENETIC MAPPING

Human genomes are 99.9% identical; only 1 letter in 1,000 is different. Although any two human genomes are thus very closely related, one genome might encode Albert Einstein while another encodes John Lennon. Genetic variations can also cause cancer in one person and let another live to the age of 120. The differences matter! Genetic mapping uses variations in genomes to approximately locate a gene on a chromosome without knowing its precise identity.

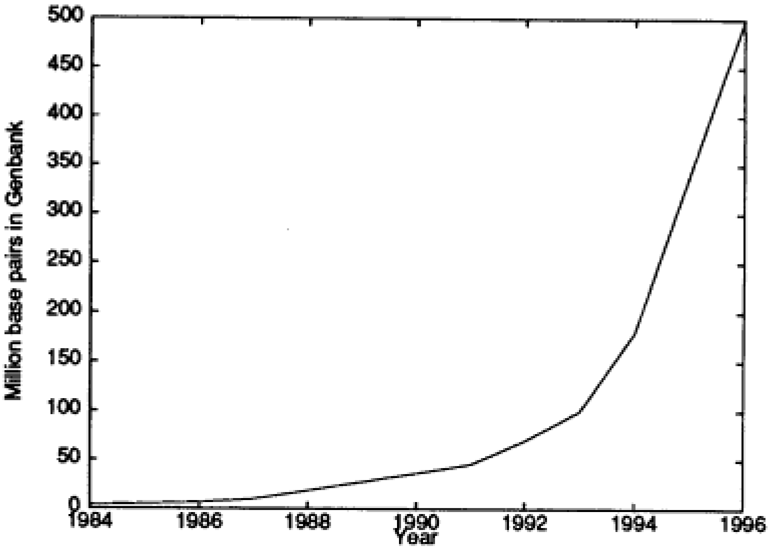

Figure 1.
Growth from 1984 to 1996 of the number of human genome base pairs (in millions) in the GenBank DNA database. Source: National Center for Biotechnology Information, available online at <www.ncbi.nlm.nih.gov> through the release notes at <ncbi.nlm.nih.gov/genbank/gbrel.txt>.

Our model is Mendelian genetics, where each individual has chromosomes in pairs. An offspring gets its chromosomes from its parents, and each parent transmits one of its chromosomes at random. Look at Figure 2, where at two locations (loci) there are genes A and B that have distinct versions on each of the chromosomes, designated by different shades. If this was all that happened, we as a population of organisms would not be so interesting and varied. But according to a random process, the pair of chromosomes cross over, giving novel associations of the regions A and B (see Figure 3). This greatly enriches the variety in the population.

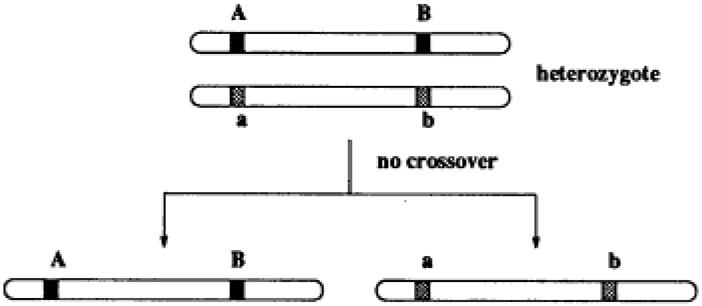

Figure 2.
Inheritance of genes when there is no crossover (mixing) between chromosomes from the two parents.

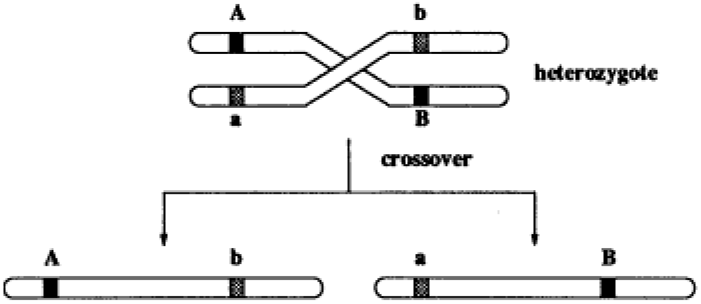

Figure 3.
Inheritance of genes when there is crossover (mixing) between chromosomes from the two parents.

Genetic mapping is founded on the idea that the fraction of times that the dark A and B alleles are together in the offspring can be used to estimate how close the genes A and B are on the chromosome. The first genetic map showed the relative locations of six genes in the fruit fly and was constructed in 1911 by Sturtevant when he was a sophomore at Columbia University. The map was built without knowing that chromosomes and genes were DNA. From one point of view, genetic mapping is a statistical problem.

In 1980 David Botstein started the modern era of genetic mapping with the recognition that naturally occurring variation in DNA can be used for mapping. Until then only genes with changes that resulted in observable differences in offspring could be mapped. Using Botstein's idea, many sites along the genome can be mapped. When a disease gene, for example, is near a mapped DNA location, the geneticist can then learn the approximate location of the disease gene. This requires having a fairly dense set of so-called DNA markers. Genes ranging from those for cystic fibrosis to the so-called fat gene have been mapped, and mapping is part of the discovery process that is so often in the headlines of our newspapers. By 1994 there were 5,826 loci mapped in an international collaboration. This represents a marker for approximately every 700,000 nucleotides along the human genome.

The mathematical issues are statistical as the problem is one of estimation of marker order, and the location of a disease gene is of statistical significance. The ordering is done using a maximum likelihood statistic, and issues of algorithm efficiency are critical in these problems. Details of the affected families and of human genetics must be incorporated in order to have useful results. See the book *Analysis of Human Genetic Linkage* (Ott, 1991).

PHYSICAL MAPPING

Genetic mapping gives locations between markers in units of *genetic distance*. Genetic distance is not measured in numbers of letters, and although it is obviously valuable to map a gene, what next? Biologists then make physical maps of large regions of DNA, even as large as chromosomes, by constructing overlapping sets of intervals of DNA called clones. DNA in the clones can be manipulated easily (whereas chromosomes cannot), and they allow for more detailed characterization later such as sequencing.

Here is the most straightforward model. We wish to map the DNA in a genome [0,*G*]. We can draw *N* random samples of shorter intervals, of length *L*. For each length *L* interval, we can characterize the interval by performing experiments obtaining data that is called a fingerprint. The most detailed fingerprint is the DNA sequence, but usually the fingerprint is much less detailed. Then the fingerprints are examined for commonalities, and when overlapping clones can be identified (Figure 4), those overlaps are information that can be used for map construction. Ideally we can establish all the islands of overlapping clones from the genome.

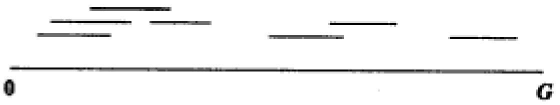

Figure 4.
Graphical depiction of overlapping clones from a genome [0,G].

With perfect data, this is just an interval graph problem, but with realistic data sets, the problem is NP-hard. Consequently, algorithms for map assembly are an interesting area for the computational biologist.

Another mathematical issue arises here, that of predicting the progress of a mapping experiment. How many clones must be examined before a given fraction of the genome is paved with clones? Here probability and statistics have been applied to study the coverage problem. The statistical results obtained are basic to the design of physical mapping experiments. See Chapter 6 of Waterman (1995).

DNA SEQUENCING

In 1980 a Nobel prize was given to Gilbert and Sanger for their 1976 inventions of two methods of reading DNA sequences. Today refinements of these methods are routinely used to read a contiguous interval or string of DNA up to 450 base pairs in length. The process of using this ability to read short substrings to determine strings that are 100 to 1,000 times longer is quite involved. First I discuss the so-called shotgun sequencing method. Shotgun sequencing has been employed since the invention of rapid sequencing in 1976 and, although there are many methods that modify this essentially random approach, it is still widely used. For our purposes shotgun sequencing is physical mapping with a one sequence as a fingerprint.

Real DNA sequence assembly has several difficulties that idealized problems do not. There is an issue about whether a clone sequence or its reverse complement has been read. More problematic is the fact that real sequence data comes with errors, in the form of mismatches, and insertions or deletions (indels). Several strategies have been developed for sequence assembly, but all have the following outline: (1) Compute fragment overlap statistics, (2) then make an approximate alignment, and (3) finally make a multiple alignment from the approximate alignment. This is clearly another problem involving algorithms and statistics. See Chapter 7 of Waterman (1995).

Recently, a new approach to sequencing DNA was proposed, sequencing by hybridization (SBH). This method might also be called sequencing by k-word composition. The idea is to build a two-dimensional grid or matrix of all $4k$ k-words. At each (i,j) is attached a distinct k-word or probe. This matrix of probes is referred to as a sequencing chip. Then a sample of the single-stranded DNA to be sequenced is presented to the matrix. This DNA is labeled with a radioactive or fluorescent material. Each k-word present in the sample is hybridized with its reverse complement in the matrix. Then, when unhybridized DNA is removed from the matrix, the hybridized k-words can be determined using a device to detect the labeled DNA.

The mathematical problem of determining sequences by SBH goes as follows: given the k-word content of a sequence, what is the sequence? The scientist's first approach was to represent the k-words as vertices of a graph with edges between vertices that shared $(k - 1)$-word prefixes and suffixes. The problem is to search for tours that visit all vertices. This is an instance of the Hamiltonian path problem, and it is another NP-complete problem. Pavel Pevzner pointed out the connection between SBH chip technology and the Koenigsberg bridge problem, reducing the problem of reading sequences to a very computable Eulerian path problem. Now the $(k$-1)-words are vertices and the k-words are edges. The length of sequences read by SBH is quite short because values of k that are practical are not much beyond 12 today. In addition the data are not precise 0-1, no-yes values. See Chapter 7 of Waterman (1995). Scientists are at work to overcome these problems.

SEQUENCE COMPARISON

Molecular biology depends on databases that contain the known sequences. Almost all sequences are entered before or within a month of publication. Rapid entry is important so that biologists can learn the relationship of new sequences with other sequences that have been determined. The Human Genome Project has increased the rate and volume of DNA sequencing. One reason for organizing biological sequences into databases is to learn new biology. Evolution can conserve useful sequence patterns over evolutionary time. When a new sequence has significant similarity with a known sequence, there is a good chance that the biological functions might also be similar. Thus, new and useful biological hypotheses are created by the results of sequence comparison.

For some notation, let $\mathbf{a} = a_1a_2...a_n$ and $\mathbf{b} = b_1b_2...b_m$ be two sequences of length n and m. An alignment is produced when null elements, —, are inserted into the sequences. The new sequences should both be of the same length L. Now write the two sequences one over the other. With the insertion of —, $\mathbf{a} = a_1a_2...a_n$ becomes $\mathbf{a}^* = a_1^* a_2^*...a_L^*$, and $\mathbf{b} = b_1b_2...b_m$ becomes $\mathbf{b}^* = b_1^*b_2^*...b_L^*$. The subsequence of $\mathbf{a}^*$ or $\mathbf{b}^*$ of those elements that are not equal to — is the original sequence. The sequence alignment is now

$$\begin{array}{l} a_1^*a_2^*\cdots a_L^* \\ b_1^*b_2^*\cdots b_L^*. \end{array}$$

Now take $s(a,b)$ to be a real-valued function for pairs of letters from the alphabet.

The similarity $S(\mathbf{a},\mathbf{b})$ of **a** and **b** is defined by

$$S(\mathbf{a},\mathbf{b}) = \max \sum_{i=1}^{L} s(a_i^*, b_i^*),$$

where the maximum is over all alignments.

The computation of the minimum over the exponential number of alignments can be found using dynamic programming in time and space $O(nm)$. This is an appropriate function to compute when two sequences have a common evolutionary ancestor, from which all letters in each sequence have been derived. In modern molecular biology, it is common to find a stretch of sequence that matches well between two otherwise unrelated sequences. This is a more complex algorithm question. First define the objective function, $H(\mathbf{a},\mathbf{b})$.

$$H(\mathbf{a},\mathbf{b}) = \max\{S(a_i a_{i+1} \cdots a_{j-1} a_j, b_k b_{k+1} \cdots b_{l-1} b_l) : 1 \le i \le j \le n, 1 \le k \le l \le m\}.$$

Define $H_{i,j}$ to be the maximum similarity of two segments ending at a_i and b_j:

$$H_{i,j} = \max\{0; S(a_x a_{x+1} \cdots a_i, b_y b_{y+1} \cdots b_j) : 1 \le x \le i, 1 \le y \le j\}.$$

A dynamic programming recursion can be given for $H_{i,j}$.

Theorem 1 For the above assumptions, also let $s(-,b) = s(a,-) = -$ for all letters a, b. Set $H_{i,0} = H_{0,j} = 0$ for Then

$$H_{i,j} = \max\left\{0, H_{i-1,j-1} + s(a_i, b_j), \max_{1\le k\le i}\{H_{i-k,j} - \delta\}, \max_{1\le l\le j}\{H_{i,j-l} - \delta\}\right\}.$$

The point of all this is that when $H(\mathbf{a},\mathbf{b})$ is greater than or equal to 0, then the maximum value of H_{ij} for $1 \le i \le n$ and $1 \le j \le m$ is equal to $H(\mathbf{a},\mathbf{b})$.

The last algorithm for H and a number of even more rapid heuristic algorithms are widely used for comparing sequences with databases. Almost any disease-gene discovery has involved running sequence comparison programs to locate the critical relationships from among millions of potential ones. See Chapter 9 of Waterman (1995).

This leads directly to another mathematical question. Since we are making millions of sequence comparisons, how do we locate the statistically significant ones from those that are just the result of the extremes of comparisons of sequences related only randomly? The answer involves some pretty probability and statistics, including use of Poisson approximation via the Chen-Stein method. Statistical methods are critical to the biological discoveries as well as algorithms. See Chapter 11 of Waterman (1995).

EVOLUTIONARY BIOLOGY

It might not be evident to a non-biologist that evolution has already been discussed in this short manuscript. Genetic mapping today is founded on knowledge of Mendelian genetics and on DNA sequence structure. Sequence comparisons find valuable biological information and create useful hypotheses by locating sequence regions that might have common evolutionary history and therefore might have common structure or function. But there are fascinating questions that arise from a close look at sequences and evolution.

DNA is copied by a very accurate process, and if no errors were ever made, there would be no molecular evolution. But of course errors do occur, at a very low rate. Errors that are saved in the population are called mutations. The descendants of the ancestral individual accumulate these mutations and they diverge from the individual over time. By examining DNA sequences from a specific gene or region of the chromosome from different species or different individuals from one species, the evolutionary biologist tries to reconstruct aspects of evolutionary history. Headlines have been made by studies of DNA sequences of human mitochondrial genomes. The mutations have been used to argue for African human origins as well as estimates of the time that humans left Africa. The analyses leading to mitochondrial Adam and Eve are based on a collection of statistical, algorithmic, and heuristic arguments.

A beautiful stochastic process, the coalescent, introduced by John Kingman (1980), arises as an approximation to many models of reproduction in genetic systems. See also Tavaré (1984). The genealogy of a sample of n DNA sequences (which we take to be aligned sequences) from a large population is described in terms of independent exponential random variables T_n, T_{n-1}, ..., T_2. The length of time to the first common ancestor (at which time there are now n-1 common ancestors) has an exponential distribution with parameter $n\ (n - 1)/2$. Two of the individuals are chosen to coalesce, and the process is run again independently with n-1 individuals to find the second coalescent time, until there is only one "individual" left, and we have found the most recent common ancestor. This time in units of N generations, where the effective population size is N, is $2(1\text{-}1/n)$. Many useful and pretty results come from this model and its generalizations, including the Ewen's sampling formula.

A wide collection of problems exist in connection with sorting molecular evolution out. These include more stochastic processes on trees where a mathematically solid model for inserted or deleted letters has not yet been formulated. In addition, trees that "best fit" a collection of molecular sequences are inferred by a variety of methods and theories based on the resulting trees, although usually only heuristic methods have been employed. See Chapter 14 of Waterman (1995).

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Mathematicians as Educators

Hyman Bass
Adrain Professor of Mathematics Columbia University

The mathematical sciences professions are in a phase transition, from which they may well emerge smaller and/or redistributed and much more dispersed. We are not an endangered species, but our health depends on being able to transcend our historic tendencies toward insularity, on our outreach to all of our sister and client communities. This message, in diverse forms, is widely heard today.

The internal mathematical culture continues its deep investigation of the fundamental structures of number, space, dynamics, . . . now with the added exploratory and processing power of new technology. These investigations are guided partly by purely intellectual evolution, but largely also by the natural sciences, to which mathematics furnishes the language and concepts for description, analysis, modeling, simulation, and other efforts. In addition, mathematics provides design and simulation tools for engineering, technology, and for the organization and decision processes of industry. These diverse functions of mathematical thinking and tools are increasingly manifest in many professions, and across the technical work force.

The phase transition mentioned above involves many partial shifts of focus—from core mathematics toward applications and toward interdisciplinary work with the natural and social sciences, from academic to industrial and laboratory settings, from individual self-directed work to collaborative and multidisciplinary efforts, from technical communication with co-specialists to translational communication across disciplinary and cultural boundaries, and so on.

Mathematics education is fashioned to provide appropriate mathematical knowledge, understanding, and skills to diverse student populations. At the post-secondary level, such education is entrusted to two large communities. One is based in our system of 2-year and community colleges. The other consists of academic mathematical scientists, most of whom have been trained principally to do mathematical research, but for whom the economic base of their profession is now predominantly this educational mission. There has also been a small but distinguished group of scholars doing research and curriculum development in post-secondary mathematics, in the tradition of Polya—for example, Ed Dubinsky, Joan Ferrini-Mundy, Steve Monk, and Alan Schoenfeld.

The shifts described above are reflected in corresponding profound changes in the role of mathematics education. In the post-WWII years we had designed a powerful educational model for producing an elite cadre of highly trained and motivated students destined for sophisticated scientific and technical careers. Some very able and committed mathematicians turned their professional energy to this educational task, often with inspiring success. But for the most part the pedagogy was formal, didactic, often brilliant, and often severe. The many students whom it alienated and whom it filtered out of advanced mathematical study were deemed to fail the high standards of our calling. They were seen as lacking "the right stuff." Since the country did not

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Note: The author emphasizes having greatly benefited from discussion of these ideas with Deborah Ball, Joan Ferrini-Mundy, Hugo Rossi, and Lynn Steen.

require vast numbers of mathematically trained professionals, and there was sufficient mathematical talent and motivation to survive any pedagogy, this filtering system was considered benign. Many even considered it desirable.

The emergence of a highly competitive and technological world economy has fundamentally enlarged the demands on mathematics education. We now seek, for the broad work force, levels of scientific and technical competence and literacy that approach what was formerly deemed appropriate for only a select and specialized student population. These same changes make increased demands of technical literacy for responsible and informed participation in our modern democratic society. These pressures give an added practical edge to the traditional argument for the cultural enrichment and intellectual empowerment that mathematical ideas and thinking can confer. When large numbers of students fail and/or leave mathematical study, which is the gateway to such competence and literacy, this is judged now to be the failure not of the students but rather of the educational system. Moreover, the students lost are disproportionately from the minority and female populations that constitute the major influx into the work force.

The time has come for mathematical scientists to reconsider their role as educators. We constitute a profession that prides itself on professionalism, on an ethos of quality performance and rigorous accountability. Yet academic mathematical scientists, who typically spend at least half of their professional lives teaching, receive virtually no professional preparation or development as educators, apart from the role models of their mentors. Imagine learning to sing arias simply by attending operas; learning to cook by eating; learning to write by reading. Much of the art of teaching—the thinking, the dynamic observations, and the judgments of an accomplished teacher—is invisible to the outside observer. And, in any case, most academic mathematical scientists rarely have occasion to observe really good undergraduate teaching.

While one does not learn good cooking by eating, neither does one learn it just by reading cook books or listening to lectures. Cooking is best learned by cooking, with the mentorship of an accomplished cook, that is, by an apprenticeship model. In fact, teacher education also is designed with a mixture of didactic and apprenticeship instruction. Professional development of academic mathematical scientists as teachers should perhaps be similarly modeled on learning in the context of practice, with only relatively small doses of more formalized styles of learning with which we are most familiar. Good designs for doing this in a systematic way are not now common. Education professionals can help us in creating and experimenting with such designs.

Effective teaching requires that teachers know their students, to be able not only to explain things to them, but also to listen to them, closely, and with understanding. And knowing something for oneself, or for communication to an expert colleague, is not the same as knowing it for explanation to a student. Further, the experience of a mathematical scientist as a learner may not be the best model for the learning of his/her student. These are the kinds of skills and awarenesses that professional development can help cultivate.

Of course there have always been in our professional ranks some very effective, even inspiring, teachers. They have become so through a combination of talent, personal commitment, hard work, and practice—and without recourse to professional educators. But do these isolated individuals constitute a model for the educational responsibility of our profession? Are we—and the public we serve—to be content with the condition that some few among our ranks have chosen to take the individual initiative to develop their teaching skills? Imagine, by contrast, abandoning our disciplined education in rigorous mathematics for future researchers to a laissezfaire system of individual self-instruction, when the impulse happens to be present. How might that affect the quality of our research community?

The disposition of many mathematicians toward the problems of education well reflects their professional culture, which implicitly demeans the importance and substance of pedagogy. Mathematical scientists typically address educational issues exclusively in terms of subject matter content and technical skills, with the "solution" taking the form of new curriculum materials. Curriculum is, indeed, a crucial aspect of the problem, and one to which mathematically trained professionals have a great deal of value to offer. But taken alone it can, and often does, ignore issues of cognition and learning, of multiple strategies for active engagement of students with the mathematics and for assessing their learning and understanding. Ironically, the mathematical preparation of school teachers is frequently entrusted to these same mathematical scientists, who are often neither trained in nor sensitive to the pedagogical aspects of teaching mathematics to young students. Pedagogy is not something to be added, after the fact, to content. Pedagogy and content are inextricably interwoven in effective teaching. Pedagogy, like language itself, can either liberate or imprison ideas, inspire or suffocate constructive thinking.

In fact, change on this front has already begun to occur, most notably that stimulated by the so-called calculus reform movement. (For an excellent report on this, see *Assessing Calculus Reform Efforts* by Alan Tucker, Mathematical Association of America, Washington, D.C., 1995.) To the often skeptical mathematicians outside this activity, the phenomenon is seen as one producing new curricular materials, and introducing much more systematic uses of technology in teaching calculus. These new materials have been the subject of animated and healthy debate, although some of the opponents have been so stridently and indiscriminately critical as to polarize discussion and impede rational discourse. On the other hand, the people actually engaged in reform calculus teaching typically have a different sense of its significance. They show the same healthy skepticism toward curriculum materials that mathematical scientists have always shown, and they exercise appropriate professional judgment on the manner and extent of use of these materials. What they find most significant about the reform is their personal transformation and the change in their professional practice as teachers. They gain a sense of having become members of a community for which the practice of teaching has become a part of professional consciousness and collegial communication, not unlike their professional practice of mathematics itself. It is the creation of this substantial community of professional mathematician-educators that is, to my mind, the most significant (and perhaps least anticipated) product of the calculus reform movement. This is an achievement of which our community can be justly proud, and which deserves to be nurtured and enhanced. In addition to the ACRE report cited above, the Joint Policy Board on Mathematics study *Rewards and Recognition in the Mathematical Sciences* is an important gesture in this direction, one that is widely appreciated and cited by our colleagues in other disciplines.

Some might be inclined to cite the calculus reform movement as a case of teaching improvement without the aid of professional educators. On the contrary, there were instances of significant consultation with education specialists. Moreover, the mathematicians who were fully engaged in this from the early stages, and who had to design programs to prepare the teaching staffs for these new courses, effectively became education specialists with a particular kind of professional expertise. They were funded, and they devoted a major part of their time to this development. (I do not rule out the possibility that an education professional may also be a mathematician.) Furthermore, it is quite evident that the pedagogical philosophy that guided the calculus reform powerfully reflected that expressed in the K-12 reform efforts, which came from the thinking of the professional education community.

Once persuaded of the need for improved professional development as teachers, as many mathematical scientists and/or departments (often under external pressure) have become, how do we go about achieving this? How, without prior professional development as teachers, can we mathematical scientists design courses and/or programs to provide this now, for present and future faculty? Part of the answer is that we cannot do it alone, either as mathematical scientists isolated from experienced professional educators (who may themselves also be mathematically trained), or as individual mathematical scientists without the collective support of our ambient departments and institutional environments. Many mathematical scientists have tended to look upon education professionals with doubts bordering on ill-disguised contempt; it is not an easy proposition that we now have much to learn from them and need their professional help. Much remains to be done to establish contexts for respectful communication and professional collaboration between mathematical scientists and education professionals—from school teachers to people doing education research. This is ultimately a two-way street, along which mathematical scientists can contribute to the disciplinary strengthening of school programs and teaching practice, while the teacher and education research communities can elevate the pedagogical consciousness and competence of academic mathematical scientists.

Mathematics education, unlike mathematics itself, is not an exact science; it is much more empirical, and inherently multidisciplinary. Its aims are not intellectual closure, but helping other human beings, with all of the uncertainty and tentativeness that that entails. It is a social science, with its own standards for evidence, methods of argumentation and theory building, professional discourse, and so on. It has an established research base, from which a great deal has been learned in the past few decades that has an important bearing on the educational performance for which academic mathematicians are responsible.

What kinds of things need to be done? At the very least, our graduate students, who regularly perform as teaching assistants or instructors, must be given serious teaching preparation, not only for their duties while they are graduate students, but also for their roles as possible future university or college faculty, or even as school teachers. Even if their career paths do not take them into the academic world, much of what they need to learn in the way of teaching skills forms part of the broader need for better communication skills in diverse settings. This will make them better and more effective spokespersons in their work and communities for the importance of sound mathematics education. As part of the general aim to make our graduate students more professionally versatile, the professional development of teaching and communication skills is a vital component. Indeed, such professional development is appropriate for current mathematics faculty as well as for graduate students. In addition, mathematics education provides an important option in the design of new professional master's degree programs in mathematical sciences departments. The resources that support such programs should also provide for the ongoing professional educational development of current faculty.

A further important challenge is the design, by mathematical scientists in collaboration with education professionals, of mathematics courses, based in mathematics departments and devoted to the mathematical preparation of future school teachers. Of course one must distinguish here the needs of elementary teachers from those of secondary teachers. This is an area desperately in need of thoughtful development and experimentation, and which has not received the quality attention by mathematical scientists that it deserves. It invites the possibility of some novel and creative collaborations, where conventional ways of thinking have repeatedly failed to produce desired results.

The above kinds of efforts can be greatly facilitated by networking with colleagues on other campuses, where similar efforts are more highly evolved. There are various activities organized by the Mathematics Education Reform (MER) network and in special sessions at the winter joint meeting of he Mathematical Association of America/American Mathematical Society that support such networking.

While mathematics and mathematics education in the United States at the school, college, and graduate levels have historically been culturally and professionally separated—a separation visible in the distinct agendas and cultures of the AMS, MAA, American Mathematical Association of Two-Year Colleges, and National Council of Teachers of Mathematics—it becomes clear to anyone who contemplates the needs for improvement of mathematics education in the United States that this problem cannot be realistically segmented into components for which these four communities take separate and uncoordinated responsibility. As mathematical scientists, as mathematics education researchers, and as teachers in universities, colleges, community colleges, and schools, we must begin to see our concerns for graduate, undergraduate, and K-12 education as parts of an integrated educational enterprise, in which we have to learn to communicate and collaborate across cultural, disciplinary, and institutional borders, just as we are called upon to do in mathematical sciences research.

Educating Mathematical Sciences Graduate Students

Ronald G. Douglas
Executive Vice President and Provost Texas A &M University

Contemplating changes, especially the possible change of a system that has worked so well for the people involved, can produce more anxiety than change itself. Despite evidence that something is wrong, that circumstances are different, our gut reaction is to stand fast, to defend the system and the values in which we believe. In a nutshell, I think this describes the current stance of the mathematical sciences community toward graduate education.

American mathematics has enjoyed a golden age during the last few decades. Students and researchers from around the world have flocked to the United States to study and work. Many of the very best mathematicians have been turned out by doctoral programs in the multitude of fine and outstanding mathematics departments in U.S. universities. If only the federal government provided more funding to mathematicians to pursue their research. If only U.S. high school students Were better prepared for collegiate mathematics. If only more and brighter American students studied mathematics at the undergraduate and graduate level. If only public appreciation and understanding of mathematics were greater. If only there were more university jobs for young mathematicians. But these are the *only* problems and they have all been brought about by forces external to the mathematical sciences, and the solutions rest outside the community also. Or so we want to believe.

In this paper, I discuss some of the choices facing graduate education in the mathematical sciences today, presenting arguments in each case on both sides of an issue. While my personal opinions may be discernible, it is not my intention to be prescriptive. However, I do intend to be provocative and help the community confront and grapple with some of the hard decisions it faces and strike at its core values.

Before getting started let me make a few historical comments. Most American mathematicians now belong to the post-Sputnik generation. The response to this event transformed U.S. universities and academic mathematics. When I was a graduate student, the system was already in flux, but I still had some experience with the way things had been and, with difficulty, I can recall those earlier circumstances. Most mathematicians today have no such memory at all. I state this not to pine for the "good old days" but to make certain we recall that, although mathematics has a history which goes back several millennia, the current system has been in place just a few decades. Thus the notion that change will bring down the temple is probably an overstatement.

Let me now address several fundamental issues. The first concerns the size of the graduate enterprise. In particular, are there too many graduate students in the mathematical sciences? Are too many people earning doctorates?

These questions can be analyzed from various points of view. First, does the number of students exceed the capacity of the system; that is, are there students with unqualified advisors or in marginal programs? I think most people would agree that the contrary is true. There are good mathematicians without doctoral students, and many strong departments have decreased the size of their graduate programs in recent years. Some people would argue that it might be better for students if some of the very small programs were eliminated, leaving only those with a critical mass in faculty and students. Although practices in some sciences tend to encourage such an outcome, few mathematicians are prepared to argue in favor of this approach, believing that such a system would be elitist; and, in any case, almost no one is willing to support steps that would bring it about.

One important reason departments cling to doctoral programs even though the programs might be marginal may be related to the different working conditions that usually prevail for departments with and without such programs. Although cogent arguments are made linking the presence of doctoral students to the mathematical vitality of a department, the small number of students and the vanishingly small number of successful students in some departments can make one wonder about the relevance of the arguments in some cases. A second reason often concerns the role graduate students have in teaching the lion's share of lower-division mathematics at universities. However, should we decide that we need young, energetic, nonpermanent instructors and then use graduate programs to provide them?

Usually when the suggestion is made that the graduate enterprise may be too big, one has in mind the great difficulty that young mathematicians have today in finding jobs. In making this connection, two assumptions are implicit: the first is that the purpose of doctoral training is to prepare students for a university, or at least a college, position and the second is that the system has a responsibility to new doctorates to guarantee them such a job. Neither assumption has always been held in mathematics, and they are certainly not held currently in all other fields. That is why, for example, most colleagues in the humanities have little sympathy with the current plight of young scientists, since few new doctorates in fields such as history or English can expect to get an academic position. However, it is depressing to see eager, talented, young mathematicians leave the field for lack of a suitable position. Let me make one other observation. Most mathematicians strongly agree that a doctorate should be required for teaching mathematics at 4-year colleges and that it is highly desirable for teaching mathematics at 2-year colleges. However, many faculty view doctoral graduates who have taken such teaching positions as having left the community of mathematicians.

An issue related to the view of the doctorate as preparation for a job concerns the completeness of the training provided. That is, if candidates are being prepared for teaching positions, then shouldn't their graduate education reflect that? Although some recognition of this possibility is now made in most departments, I wonder what message is being given at most places on the relative importance of teaching.

Let us return to the question of size and the purpose of graduate education. Since there seems little likelihood that the number of university and college positions in mathematics will increase, at least in the foreseeable future, the profession must make some choices. Either the community must somehow reduce the size of the doctoral system, or it must broaden its views on appropriate jobs for graduates and both stimulate demand and prepare students for these alternative positions, or students must be told that one studies mathematics because one likes it. It is then the students' responsibility to find employment, and if they are fortunate, they will make use of what they have learned. Most groups that have thought about this issue have opted for the middle course, but this has consequences for graduate training. It would have to be broader and more responsive to the needs and desires of business, industry, and other disciplines. In particular, students would need to learn how to communicate what they have learned to the educated public. The realization that graduate education would have to adapt is particularly hard for many mathematicians, especially those in core areas, to accept. Part of the problem is that of uncertainty—we know how to prepare students to follow our path, to be a copy of ourselves, but how do we educate them to do something else? I will not go into detail about how to broaden graduate education, since much has already been written on that subject. Moreover, I believe that if a department becomes convinced that change is necessary and identifies the goals for its doctoral program to achieve, then an approach can be devised.

There is one other dimension that merits our attention here: the increasing length of time that students are taking to complete a doctorate. Calls to broaden graduate training can only exacerbate this trend unless hard choices are made about what is essential. There are, however, other reasons. Both advisors and students see little reason for finishing and entering an unfriendly job market; also, both want a candidate's thesis to be as competitive as possible and the curriculum vitae to list several papers. Would postdoctoral apprenticeships available to a larger share of the new doctorates reverse the trend? Would young mathematicians be both helped and encouraged to pursue realistic career choices sooner in such positions? In any case, does a longer doctoral program really benefit students or the profession in the long run?

An issue related to doctoral education is the role of professional master's programs in the mathematical sciences. For several decades, most master's degrees have been awarded as consolation prizes to students not allowed to continue toward the doctorate. There are some thriving professional programs, however, almost all in applied areas. Anecdotal evidence suggests that much of the need of business and industry for mathematically trained professionals could be supplied by well-designed professional master's programs in the mathematical sciences. Might not the best of these students, or those who are sufficiently motivated, still go on to doctoral studies?

Some people believe that the fight solution to the size problem is to raise the bar, that is, to keep the same number of programs and about the same number of students, but to make it harder to complete the doctorate. This could be done by requiring stronger dissertations or broader training. For people who view the profession as a kind of priesthood, it is appealing to reduce numbers by keeping out all but the most worthy. However, there might be several negative consequences to such an approach.

First, there would be the terrible human waste of labeling a large group of our most talented people as failures and choking them out, ill prepared for anything. Moreover, since a good many such people often find their way into policy positions or positions in which they mediate mathematics to the public, they might turn the tables on the mathematics community, strongly affecting public funding for mathematics and universities.

Second, while Darwinian selection appeals to many mathematicians as a fair way to choose who succeeds, the playing field is often not as level as many would like to believe. In many cases, it's as though someone taught some of the animals how to use weapons and then accepted the outcome of which animals survived as having been dictated by nature. That is, success in graduate school does not always go to the fittest or ablest, but often to those who are best prepared. And such preparation may have had little to do with an individual's abilities. For example, it might depend on where one was born in the United States or the economic status of one's parents. It can depend on how you were encouraged in school and whether or not your teachers expected you to excel in mathematics. There is also the possibility that students with reasonable preparation in U.S. colleges and universities might find themselves competing with students from abroad who have several years of postbaccalaureate experience.

As an aside, let me ask if graduate students in any other country must compete with the brightest and best from around the world. This policy may work to maintain the strongest mathematical sciences community in the United States, but there may be a cost. Perhaps some American students choose, as a consequence, not to compete or to drop out of graduate school early in the process. The perception may be developing in schools, colleges, and universities that studying mathematics and the sciences is for someone else. Let me acknowledge that the presence of international students and researchers has been a source of strength and vitality for American mathematics in this century and that there would be a real loss if that were changed. However, what has changed during the past two decades is the proportion of students from abroad. It is appropriate to ask whether this change has negatively affected the nature of graduate programs in many cases.

About 5 years ago I chaired a study of doctoral education for the Board on Mathematical Sciences. The findings of our committee were rather dramatic and surprising to many people. Agreement on purpose and the general environment had a substantial effect on the success of students in a graduate program. This was particularly true for women and minority students. These groups are disproportionally affected in programs that adopt a hands-off attitude and subscribe to a "survival of the fittest" philosophy.

In summary, the approach of raising the bar or allowing Darwin to prevail might have several consequences as to who studies mathematics. I think the community must decide how much the composition of the U.S. mathematical sciences community matters. Will it affect mathematics education in the schools, colleges, and universities? Will it affect the public -perception of mathematics? Will it affect public funding of the mathematical sciences?

As I indicated, my goal at the outset was to raise questions, to point out problems. Mathematicians pride themselves on rational thinking and avoiding contradictions. I believe there are some contradictions in our current policies, or at least in our actions. I hope we will work to resolve them.

A View of Major Trends at Research Universities

E.F. Infante
Senior Vice President for Academic Affairs University of Minnesota

There is no doubt about it—we find ourselves in a changed and changing environment. I am reminded of the remark that the only person who likes a change is a baby with a wet diaper, and that only so that she may return to the "initial condition." Change is painful but nevertheless necessary if we, within the academic community, are going to retain and strengthen our core, enduring values.

In this workshop, we are focusing on the mathematical sciences, on the research funding environment, and on education in mathematics. I thought it desirable to attempt to verbalize the changing environment of our universities, for they are crucial to our mathematical enterprise. The mathematical sciences, perhaps more than the other sciences, are very university-based; moreover, the ubiquitous nature of mathematics—in precollegiate education and throughout the industrial and service sectors of our economy—does not give it a center outside our research universities. To preserve the strength and appropriately meet the challenges facing the mathematical sciences requires actions that must be centered at our research universities. It is thus essential that we attempt to understand the challenges that these institutions are facing.

What I will attempt to contribute to this workshop is an articulation of the external pressures on higher education as an enterprise, and an outline of what seem to me the notable challenges, trends, and responses on the part of our universities.

EXTERNAL PRESSURES AND CHALLENGES

There seem to me to be a very large number of trends, pressures, and concerns that our society and its political and economic leadership are imposing on the higher educational establishment. A cacophony of voices, a diversity of expectations, and contradictory requests are all evident. One clear message, and I believe a positive one, is that our society well understands the importance of our higher educational institutions, and has insistent—although diverse—expectations. To better attempt to understand these, and the changed environment within which they take place, I describe them within four categories:

- The fragmentation of the stakeholders' interests in higher education;
- The change in the nature of the *demand* for university education;
- The limitations of resources (and the corresponding increase in accountability); and
- The globalization of higher education.

There is abundant evidence that the coherence of expectations of the past 50 years by the many stakeholders in higher education is badly frayed. The end of the Cold War, very serious social problems, and the changing expectations regarding an appropriate education for today's job market cause a diversity of claims by different stakeholders.

Universities are bombarded by fresh concerns about the balance between teaching, research, and service to the community. Research universities are being particularly criticized for an overemphasis on research, the questionable relevance of some research, and a lack of attention to teaching. The new language of customer orientation is used to state that the student is the customer (some of us still believe she is an investor), and that research and outreach must be focused on problems of immediate relevance. At the same time, there is a deep understanding within the leadership of our society that the benefits of our investments in research have been enormous and are the foundation of our economic strength. It is notable that, in spite of indicators that our political leadership values undergraduate education more highly than research, there continues to be a deep commitment to sustaining our university-based research enterprise. There seems to be a developing agreement among thoughtful observers that the established paradigm of teaching, research, and service, with its implication of separate and contradictory activities to be carefully balanced, is yielding to what seems to me a more descriptive paradigm of learning, transmitting, and applying knowledge. And this, I believe, is supportive of our research universities.

The fragmentation of stakeholders' interests in higher education is particularly deep between the public, elected officials, boards of trustees, and faculty of our institutions. Parents, and the public, are clamoring for access, affordability, and, increasingly, a vocational-oriented education that prepares the student for entry into the job market; the public feels squeezed financially, expects value for its investment, and demands relevance. Public officials' concerns center on effectiveness and efficiency; they share the public concern over relevance and job preparation but are conscious of the importance of investments in long-term research that addresses economic, medical, and technological needs. Internally, within the universities, faculties espouse the traditional core intellectual and academic values that on the surface seem to contradict those of the public and of their elected representatives.

This fragmentation of interests is underscored by the changing nature of the demand for university education. In orientation, values, and structure, higher education is still fundamentally based on concepts developed when participation in higher education was the privilege of a small minority of individuals in the 18- to 22-year age range. Today, participation in higher education has expanded enormously to include almost two-thirds of those in this traditional age range, plus a significant number of older, part-time students. In today's job market a college degree is becoming as necessary as a high school diploma used to be; the differences in compensation and in opportunity for advancement are striking. Furthermore, there is an increasing need for training and retraining of college-educated individuals. Thus, a focus on job-oriented education and on economic participation is driving an emphasis on relevance—indeed on what can be called *vocationalism*—and thus changing the demand structure for higher education. Let it be understood that this trend is occurring not only at the undergraduate level but also at the postbaccalaureate level, with profound effects on the support of, demand for, and appropriate preparation of participants in graduate programs.

There is no disagreement that resources in support of higher education are limited. Indeed, in some areas we have faced net decreases, in spite of increased demands. In the public sector, higher education and research are "discretionary" expenditures, corollaries to major decisions—and increased expenditures—on health care, environment and social services, and elementary education. During the next 7 years, it is expected that higher education will increase by 20%, with, at best, half of that increase matched in financial resources. It often happens that when legislatures, or Congress, have no funds to appropriate, they instead regulate and add to the accountability process. In spite of the well-deserved demise of the U.S. Department of Education's Post-secondary Review Entities, this increase in accountability and the burden it places on higher education is continuing. This is evident in regulations imposed on grant management by both the National Science Foundation and National Institutes of Health, in rules regulating faculty workload and financial aid for students, and in the use of "performance funding" mechanisms of financial support institutions of higher education.

Finally, the globalization of the economy is also having its impact on the globalization of higher education and research. We in the United States have become accustomed to playing an overwhelmingly dominant role in graduate education and research in relation to the rest of the world. Clearly, globalization implies that we cannot expect to continue to do so, and this change is particularly important for our graduate programs in the mathematical sciences.

INTERNAL RESPONSES

What are the academic community's most essential responses to these external changes? I suggest they are the following:

- Adapting to less money;
- Dealing with increased accountability;
- Utilizing technology;
- Developing novel roles and responsibilities for the faculty; and
- Developing new activities and new forms of competition and cooperation.

Our universities have been adapting to the shrinkage of resources in three stages. The first stage has consisted of what could be called "pruning plus across-the-board cuts." We have experienced reductions in faculty and administrative personnel and the elimination of very small and marginal activities (seldom, academic programs). This stage was expected to provide an appropriate bridging to a near future when resources would increase again. But resources have not increased.

The second stage, in which many of our institutions are now involved, is an adaptation of the use of market mechanisms: "*responsibility-centered management*," "*cost centers,*" and "*revenue centers" are* terms we are getting to know. These methods are meant to increase the effectiveness and efficiency of our operations through the use of incentives. There are few questions about the application of these management methodologies to the administrative and supportive structures of our universities; there is much more concern about their application to the academic core. Nevertheless, these means of adaptation to difficult times are being implemented, with some successes and many concerns. I believe it is inevitable that these forms of accountability and incentive will become pervasive; and I am equally convinced that it will take considerable wisdom on our part not to misuse these tools within the academy, where values, not costs, are the most important characteristic.

The third stage is what is commonly referred to as reengineering. We already see reengineering efforts being applied in the more financially stressed units of our universities, such as medical schools and related hospitals. The efforts of reengineering consist of radically changing the processes and structures, including the curriculum, of these units. We at the University of Minnesota are undergoing such an effort in the health sciences, with considerable anxiety, anguish, and concern. Given these trends, I believe that we must be prepared, as individual faculty members, to intelligently involve ourselves in reengineering activities. The most fundamental of our processes is the learning process; I posit that everything at a university depends on it. How does one reengineer that process? Moreover, an academic institution, especially a research university, is the quintessence of the organization for workers who are engaged in the development and dissemination of knowledge, individuals who depend on peer review and collegiality; it is not an industrial organization with easily measurable outputs and processes subject to total quality management (TQM) techniques. But we have little choice but to learn from others and adapt, and hopefully to invent methods designed to increase our effectiveness and efficiency.

These methodologies are designed to simultaneously increase effectiveness and efficiency, and thus stretch resources, and to increase accountability at the individual, departmental, and institutional levels. There is a major trend occurring in this area, through the institution of "measures" of performance and through funding mechanisms that depend on them and are meant to reward good performance. Responsibility-centered management within the university and "performance funding" at the institutional level are not the only accountability tools; others imply a plethora of rules and regulations, and of accounting systems that are increasingly burdensome but nonetheless inevitable. It is essential that, if we are going to meet the challenges with which we are presented, we learn to deal effectively with these accountability measures.

It is well known that the use of technology has had, and continues to have, a most profound impact on the effectiveness and efficiency of industrial operations. The development of the computer and of information technologies is having a similar effect on the service sector of the economy. Instrumentation, computers, and the use of technology have also profoundly affected research and advanced study; they are now beginning to have a significant effect on scholarly communications. There is an expectation that the use of information technology will also have a major impact on the efficiency, cost-effectiveness, and methodologies of the processes of learning and teaching. This is a promising development that is exciting and intellectually stimulating; thus far, however, it has in general not led to cost reductions. Actually, the opposite has occurred, resulting in major new expenditures for equipment and its maintenance, with mostly desirable new outcomes, but cost increases instead of reductions so far. Moreover, the increase in capital-labor ratio implied by these investments in a period of very limited resources has invariably resulted in a reduction of "labor," that is, in the number of faculty.

The most important trend that is occurring within universities is the changing roles and responsibilities of our faculties. No doubt our faculty and the discharge of their roles in learning and discovery are central to any meaningful change that will occur within the university. Increasingly, the faculty is seeing its primary mission as education, in contradistinction to the teaching-research-service paradigm, and thus generating a stronger level of connection, integration, and synergy among those components. In looking critically at learning and teaching, faculty are introducing at the undergraduate level what they have done so well at the graduate level, and using new organizational structures and technology to promote more participative learning at the undergraduate level. It is the faculty that is carefully reexamining courses and curricula. The emphasis in our research community on calculus is a notable example of major efforts at curriculum reform. Moreover, and most importantly, faculties' increased interest in education extends beyond the undergraduate level to high school and elementary education. This involvement in K-12 education is desperately needed, for its own sake, but also because our students are formed in that system. This trend is contributing to a profound and positive transformation in the outlook and attitudes of our faculty at research institutes, in how they prepare their graduate students with a new emphasis on teaching, and on teaching careers beyond research universities.

Finally, a profound change has occurred in the roles and responsibilities of the faculty that is resulting in a much higher level of involvement in multidisciplinary, multidepartmental work; in a joining together in "team" efforts much more now than in the past; and in the involvement of teachers, undergraduates, graduates, and faculty in joint enterprises that open disciplines, education, and research.

This change in roles and responsibilities is making it possible for new activities to be undertaken and new forms of collaboration to exist. The involvement of the faculty in normal activities directed to the K-12 system, interaction with other disciplines in undergraduate teaching, and cooperation with industry in the development of graduate degree programs in response to industrial needs are outstanding examples of the broadening of the horizons of the mathematical research community. These new activities are also bringing about novel forms of cooperation—across other institutions, industrial organizations, and other educational systems—with community colleges, high schools, and elementary schools. They are bringing new competition. U.S. universities are competing with each other more than ever before; they are also competing with other service providers. The competition is not only for students and funds, but also for ideas and opportunities.

The responses of the universities show a vigor and energy that contrast with the problems of low morale during this period of change and of discontent. It is a period in which change is also accommodated by low salaries, less than pleasant discussions of tenure and workload, and a most difficult environment for the young scholars who are being prepared for a teaching career. There are few positions for aspiring professors, few accolades for academics, and voiced unhappiness at high costs and low performance.

CONCLUDING REMARK

Change is difficult; it is particularly difficult for those of us who recall little but praise directed at research universities, and great individual opportunities for their scholars. Undoubtedly today, morale is low, we feel unappreciated, and we worry about the future.

Yet, if we look carefully, we cannot help but see, amidst the challenges, the strengths of our research universities. And if we listen carefully, we cannot help but hear a commitment from our society to ensure our effectiveness and performance. Let us study our strengths and weaknesses and the new environment and let us respond to the opportunities. The quality of our response will determine both the level of our support from society and our performance.

SUMMARY

Background and Overview

Thirty-five individuals from within and outside the mathematical sciences community, of whom a large majority constituted a broad, representative cross section of the community's leadership, participated in the May 1996 workshop "Actions for the Mathematical Sciences in the Changed Environment." In addition, 10 invited speakers presented their views to workshop participants regarding the general themes of new opportunities, the Washington environment, the view of scientists, perspectives on education, the roles and implications of national needs for the mathematical sciences, and major trends at research universities. A list of participants and the workshop agenda appear in Appendix A. Drafts of most speakers' papers (nine of which are presented above in this summary report) were provided to participants in advance of the workshop, along with a number of participants' individual position papers. Also, a copy of BMS's 1992 report *Educating Mathematical Scientists* (the "Douglas" report; NRC, 1992; available for reading on the World Wide Web at http://www.nap.edu/bookstore/pod/POD355.html), a summary report of NSF's recent workshop report *Graduate Education and Postdoctoral Training in the Mathematical and Physical Sciences* (NSF, 1996), a summary of the 1995 Committee on Science, Engineering and Public Policy (COSEPUP) report *Reshaping the Graduate Education of Scientists and Engineers* (NAS/NAE/IOM, 1995; also available online at http://www.nap.edu/readingroom/books/grad/index.html), and a photocopy of the (out of print) 1990 BMS report *Actions for Renewing U.S. Mathematical Sciences Departments* (available on the World Wide Web at http://www2.nas.edu/bms/215e.html) were sent to each participant as preparatory background for the event. Most of the speakers participated in subsequent workshop break-out group and plenary discussions.

Highlights

From the large number of issues raised in speakers' presentations, background information, and extended workshop discussion (see, for example, Small Group Discussion background in Appendix A), participants focused on several broad topics for much of their considerations, including (1) defining departmental responsibilities and setting departmental goals, (2) broadening and improving education, (3) making connections with other disciplines, (4) rethinking faculty evaluation, (5) framing funding strategies, and (6) improving communication, and offered (7) closing observations.

In those contexts, workshop discussions addressed a number of themes.

Note: This summary reflects the workshop discussions as refined by further consultation with workshop participants, and subsequent NRC review.

DEFINING DEPARTMENTAL RESPONSIBILITIES

- *Diversity in the focus of different mathematical sciences departments is desirable and healthy, but each department will necessarily define its unique role*. An individual department is the best body to decide what it could or should do to meet its societal responsibilities and respond to the challenges that face the community.
- *This is a good time for a department to re-examine its responsibilities and goals within its local university framework*. In so doing, it might delineate for itself what challenges it needs to address, and devise strategies for addressing them. Added to the omnipresent challenge of preserving the nation's pre-eminent strength in research and discovery in all mathematical sciences areas, there now are new challenges confronting mathematical sciences departments. They include educating large numbers of non-mathematical sciences majors; making the mathematical sciences more accessible to all students; training mathematical sciences majors in a manner that promotes career development in teaching, business and industry, government, or advanced study and research; and preparing mathematical sciences graduate students for diverse career options. Provided with the necessary resources, many departments might be even more successful in meeting these challenges than previously. Those departments that wish to have a way of demonstrating their progress, both for their own benefit and for communicating their accomplishments to others, might identify ways to assess performance or progress toward meeting these challenges.
- *Mathematical sciences professional societies are the best stewards to develop an effective means for accumulating, centrally depositing, and making available for general use (say, through the World Wide Web) valuable data of interest to many departments, as well as to the whole profession*. In recent years, a number of mathematical sciences departments have developed creative or more effective ways of performing customary tasks and implementing innovative directions or methods.[1] However, it takes considerable time for the larger mathematical sciences community to learn what works well, what has been or is being explored, and what changes are under way at various different institutions. Information about innovative departmental strategies—for instance, in curriculum and educational programs, as well as about unusually successful traditional ones—might be made more widely available as examples to emulate and analyze. A coordinated database of departmental strategies that are submitted in a standard format could be created at a single site on the World Wide Web, and availability of this database made known at national professional society meetings. Having such information posted in a consistent format on the World Wide Web would permit searches, an accumulation of data on mathematical sciences departments for continuing analysis, and a repository for archiving and accessing the experiences of different departments concerning innovation and experimental approaches in research, scholarship, and education. Such postings might also be

attractive advertising for a department, and if available in a universal format could be very useful for departments in other institutions and the whole discipline.

BROADENING AND IMPROVING EDUCATION

- *Key tasks for mathematical sciences departments are to provide undergraduate-level mathematical sciences courses that give all students an understanding of both the historical and current role of the mathematical sciences in society, that supply the mathematical understanding and skills students need to make informed decisions and to function in modern technological society, and that enhance students' appreciation of mathematics*. Mathematical literacy has become increasingly necessary in many areas of society, most especially in industry and business.[2] Since the mathematical sciences are broadly used throughout the economy, their importance continues to grow for an ever larger cross section of the population.[3] Raising mathematical skills and awareness is a long-term effort to be pursued at all educational levels, including K-12, undergraduate, and graduate. Areas here include improving the mathematical preparation of school teachers by, for instance, working with the K-12 community in designing mathematical sciences courses for potential teachers. Courses can also be designed for students who are not mathematical sciences majors that convey the importance, power, and value of the discipline in modem society. All students benefit from instruction in quantitative reasoning and problem solving, and by learning what roles the mathematical sciences play in daily events as well as in modern science, medicine, and technology. Citizens who learn about and understand the mathematical sciences enhance their own and the nation's economic capabilities. "General education" courses are a quite effective avenue for communicating examples to the educated general public that show why the mathematical sciences are crucial for the nation, how they contribute in vital ways, and what highly leveraged and far-reaching societal benefits accrue from them.
- *Employment difficulties that many advanced mathematical sciences degree recipients have recently encountered (see, for example, Fulton (1996) and Benkoski et al. (1996)) suggest it might in some cases be beneficial to adjust the postbaccalaureate education program.* Several studies and reports have raised concerns as to whether the median time to degree has become too long, and have called for adjustments in the way mathematical sciences graduate programs prepare students. For instance, the National Research Council's "Douglas" report, *Educating Mathematical Scientists: Doctoral Study and the Postdoctoral Experience in the United States* (NRC, 1992), called for broad academic preparation of mathematical scientists, and (in an appendix) advised potential graduate students in the mathematical sciences to learn, among other

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[1] Examples include the University of Chicago mathematics department's new master's degree program in financial mathematics, and the University of Notre Dame mathematics department's new policy to have senior faculty who are the most effective lecturers teach the freshman-level courses for non-mathematics majors.

[2] For example, in the communications industry, computing, much of the managerial sector, and the scientific and technological professions.

[3] Technicians and manufacturing production supervisors, for instance, now increasingly deal in quantitative issues, logical thinking, and problem solving, and so need to be more mathematically prepared and numerically literate than in the past.

things, what is the average length of time to a doctorate for a prospective mathematical sciences doctoral program. The report of a 1995 National Science Foundation (NSF) workshop, *Graduate Education and Postdoctoral Training in the Mathematical and Physical Sciences* (NSF, 1996), recommended that graduate departments broaden their educational programs by introducing more interdisciplinary courses and off-campus experience, and discussed the time to degree issue. The *Mathematics in Industry* report produced by the Society for Industrial and Applied Mathematics (SIAM, 1995) includes a number of other useful references.

- *The strength of America's graduate mathematical sciences enterprise is the envy of the world, and there is widespread, strong agreement that this strength must be carefully safeguarded whenever adjustments are made to it*. Melding breadth—which reasonably involves the general skills used in employment settings (be they academic, commercial, industrial, or educational) where most of a program's graduates secure employment—with that strength, while striving to reduce the time required to earn a Ph.D. degree, is a difficult and major issue. Resolving the breadth-vs.-length-of-program issue might depend on additional resources being available or different infrastructure being put into place in order to not diminish a program's existing strengths. One possible time to address this issue might be when a department with a doctoral program undergoes external review. One approach might be offering mathematical sciences degrees other than the Ph.D., for instance, professionally respected master's degrees that business and industry value.[4] Many successful master's degree and internship programs take advantage of geographically neighboring opportunities and local resources that are particular to an individual mathematical sciences department. A professional master's degree program may be appropriate, for instance, when a clear career path exists in the local industrial or business community for recipients of such a degree, and when the requisite departmental resources and infrastructure are available. Such an approach might help counter a potential national contraction of mathematical sciences graduate programs that informal indications suggest may be under way. Yet another possibility is to move toward mentored postdoctoral/internship experiences as a normal part of the professional development of mathematical sciences Ph.D.s.
- *One responsibility of the mathematical sciences community is to preserve and strengthen the mathematical sciences enterprise, including the research enterprise*. However, serious concerns have been expressed that, while university mathematical sciences departments are and have been doing an outstanding job of discharging all their many responsibilities, they are now being expected to do even more while not being provided the additional resources or infrastructure needed. In response to these difficult circumstances, some departments are, for example, investigating the economic costs of swapping, two-for-one, graduate teaching assistants (TAs) for mentored postdoctoral positions, where the postdoctoral teaching load would equal that of two graduate TAs. Others are determining what kinds of employment their

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[4] For practical information on starting an industrial master's degree program—and how in general to develop involvement with industry—see, for instance, Friedman and Lavery (1993) and SIAM (1995).

Ph.D. graduates are finding, and making available within their doctoral program opportunities for persons pursuing those careers.

- *Is the quality of education affected by the practice of having mathematical sciences courses in two- and four-year academic departments taught by individuals other than full-time mathematical sciences faculty?* There is at this point only anecdotal evidence that economic pressures have prompted some mathematical sciences departments to staff courses with larger numbers of adjuncts or other individuals who are not full-time faculty. One result could be an increase in the departmental tasks and responsibilities that full-time faculty members must shoulder—advising students, serving on committees, developing curriculum, handling department administration, mentoring majors, and the like. The causes and effects associated with increased reliance on non-full-time faculty, especially the effects on quality of instruction, are unclear. It is worth investigating how widespread it is, and how to best address concerns associated with the practice.
- *Because the mathematical sciences are more important for society in a wider variety of ways than at any previous time, and because the mathematical sciences profession needs to be continuously strengthened and replenished by drawing well-educated individuals from the widest possible pool of talent, it is essential to ensure that women, underrepresented minorities,*[5] *and students from educationally deprived backgrounds are encouraged both to study the mathematical sciences at all levels and to become part of the mathematical sciences community.*[6] Considerable progress has been made in recent years in attracting, retaining, matriculating, and employing a broader cross section of society in the mathematical sciences compared to former years. For example (Fulton, 1996), there are more women faculty in mathematical sciences departments than ever before (although their distribution among institutions is not yet uniform). To ensure continuing progress, particularly with respect to underrepresented minorities and educationally deprived students, approaches that work, both those with proven success and fresh approaches, need to be sustained, expanded, and replicated in other locations. One way to help propagate success might be for mathematical sciences professional societies to make broadly available from a central repository, for example, at a single site on the World Wide Web, information on what has succeeded.

MAKING CONNECTIONS WITH OTHER DISCIPLINES

- *While there has been a steady increase in mathematical sciences work linked to most major national needs as well as to almost every area of science, technology, and medicine (see, for instance, NRC (1991, 1993, 1995, 1996)), there is not yet sufficient professional recognition of these achievements, and the community is not well enough*

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[5] These include, for example, African Americans, Native Americans, and Hispanic Americans.

[6] Workshop participants noted that, in the near future, a separate workshop might propose explicit mechanisms addressing the issue of participation in the mathematical sciences.

informed about them. Sometimes the mathematical ideas involved in interdisciplinary research are very recent mathematical achievements, such as the concepts of group theory in coding and communication, or the theory of wavelets. Other times they evolve out of applied or interdisciplinary work that feeds back into the mathematical sciences and becomes general mathematical knowledge. This has recently occurred in four-dimensional geometry and mathematical physics, between topology or statistics and the Human Genome Project, and between various mathematical areas (including topology, complex analysis, probability and Statistics, and measure theory) and fractals in analysis, compression of data, and nonlinear dynamics. Mathematical sciences students at all levels should be as aware of such interdisciplinary progress as they are of efforts within their own discipline. Although a number of mathematical sciences programs offer interdisciplinary courses at all levels cooperatively with other departments, many more could do so. To involve existing mathematical sciences faculty in these courses, professional societies could periodically offer appropriate short courses to broaden the capacity of such faculty (analogous to what was done in the 1970s to address a need for faculty with computer science training). Mathematical sciences degree holders will be much better ambassadors for their discipline if they are acquainted with an understanding of what interdisciplinary work involves and an appreciation of the value their discipline brings to other areas. Mathematical sciences professional societies might establish awards and prizes to recognize interdisciplinary achievement by mathematical scientists, and encourage mathematical scientists who participate in outstanding interdisciplinary research to publish articles on their work for the mathematical sciences community.

RETHINKING FACULTY EVALUATION

- *A pivotal issue that strongly affects the future of the mathematical sciences community is evaluation of its academic members*. The activities, attitudes, and attributes deemed valuable in an academic setting set a tone that is reflected throughout the community. Since most mathematical sciences departments now function in circumstances quite different from those of 30, 20, or even 10 years ago, faculty evaluation criteria may need to evolve for a department to appropriately and successfully address current and future expectations, responsibilities, and goals. Due to the changing environment, some departments may change their hiring practices, for example, hiring faculty who pursue interdisciplinary research or mathematicians who combine an interest in curriculum development and innovative instruction with research in the mathematical sciences. Some workshop participants felt that there may be a need to rethink the ways in which mathematical sciences faculty are evaluated for promotion, tenure, and salary increases, echoing issues raised in the Joint Policy Board for Mathematics' report *Recognition and Rewards in the Mathematical Sciences* (JPBM, 1994). If new interdisciplinary areas are incorporated into a department's goals, then that department needs to adopt criteria suitable for these areas. With an increased emphasis on quality of instruction, and the importance of mentoring minority and women students, reliable ways to assess teaching effectiveness may be needed. Professional societies can be catalysts for this rethinking by identifying and widely communicating case studies and findings of proven practical techniques and approaches. Any evolution in faculty evaluation needs to strengthen a department. All parties involved (including the mathematical sciences faculty—specially new faculty—and university administrators, and the faculties of other departments) have to clearly understand what is changing and the reasons why. Since the issue of faculty evaluation has broad repercussions for the entire community, the various opinions, developments, and associated impacts need to be publicly discussed and widely communicated for the benefit of all community members. For example, panel discussions at annual meetings might be devoted to faculty evaluation in a changing environment, and might involve both faculty and administrators. There could then be follow-up reports on the discussions in professional society news publications.

FRAMING FUNDING STRATEGIES

- *The present outlook for mathematical sciences funding is one of constraint and possibly diminishment. Furthermore, previous National Science Foundation disciplinary advisory boards no longer exist due to a 1993 Executive Order. To ensure the continuing health of the discipline, most workshop participants considered it crucial that a group of representatives of the discipline be designated to work with funding agencies, particularly the National Science Foundation, in judging the adequacy of research support for the mathematical sciences, and in developing strategies to make the best use of available funds*. Many participants felt that mathematical sciences professional societies could help the National Science Foundation and other agencies make the case for the mathematical sciences based on national needs by establishing such a balanced, representative, formal group to liaise with funding agencies, and to routinely supply them with high-quality, thoughtful community input for that demanding and delicate decision making. Strategies for innovative funding might be designed to stimulate and support the mathematical sciences community's goals in research (both within the mathematical sciences and in interdisciplinary areas), in education, and in outreach programs. Contributing to this decision-making process will manifestly increase the mathematical sciences community's credibility, recognition, respect, and influence with government leaders.

IMPROVING COMMUNICATION

- *The mathematical sciences community needs to convey the excitement and importance of the mathematical sciences to the public*. When new results and their potential ramifications are announced within the community, they could also be communicated to the general public at a level that is broadly understandable by nonspecialists. Regular, but carefully prepared and presented, communications could be specifically tailored for that general public to impart a modicum of understanding, a sense of appreciation, and an impression of how the mathematical sciences contribute to the benefit of society. The value of the discipline's research and application achievements needs to be expressed to the public in terms the public will understand. Efforts in these directions such as, for example, the National Academy of Sciences' *Beyond Discovery* series (see its Web page at http:/www2.nas.edu/bsi/) or the American Institute of Physics' *WonderScience* series (see its Web page at http://www.acs.org/edugen2/education/ws/ws.htm) need to be emulated by the mathematical sciences community. The use of imperfect analogies can be very useful in this. Because of the importance of public communication efforts, mathematical sciences professional societies might identify and publicize possible sources of support for such activities. A "high-leverage" continuing opportunity is offered by entry-level undergraduate courses, which often provide the sole chance to expose the full panorama of the mathematical sciences to a majority of the general public. To significantly improve the teaching of and the effective communication with non-specialists will require concerted efforts by individuals from all sectors of the mathematical sciences community.

A number of examples were discussed by workshop participants of possible ways to improve the discipline's communication with the public, some of which are already being acted on by certain members of the community. They include:

— Having senior mathematical scientists routinely write expository and/or survey articles about accomplishments and breakthroughs in mathematical sciences research or education. These articles could target different sorts of audiences, ranging from junior high school students through mathematical sciences Ph.D.-holders, as well as the public at large.
— Having members of the mathematical sciences community collect, present, and publish expository essays, written at appropriate levels, that trace the impact of the mathematical sciences and vignettes on their role in new developments as well as in history.
— In all communication venues, emphasizing aspects of the mathematical experience that involve human interest, utilizing television and videos whenever possible, and providing role models from underrepresented groups.
— Interacting with the local community, for example, by giving talks at schools, working with teachers in developing classroom materials, initiating a teacher-exchange program, or inviting business and industry representatives to present problems to mathematical sciences groups.
— Posting information on the World Wide Web of interest to and presented at an appropriate level for the general public, and publicizing its availability.
— Devising mathematical computer games or other captivating or entertaining software based on aspects of the mathematical sciences.

— Attending and giving intra- and interdepartmental colloquia, which are an essential means of communicating the mathematical sciences to colleagues, to other disciplines, and to the public.
— Creating a variety of courses to raise mathematical awareness for non-mathematical sciences majors, as well as other courses addressing mathematical literacy.
— Creating and taking advantage of opportunities to serve in policy-making positions or in other positions that influence public policy; and
— Visiting local, state, and federal legislators, but only after preparing for such a visit by using (say) the Joint Policy Board for Mathematics' legislator-visit kit.

Closing Observations

- *To preserve the discipline's strengths while meeting challenges posed by a changed environment, the U.S. mathematical sciences community's major national responsibilities include*:

1. Promoting research and discovery in all areas of the mathematical sciences so that the U.S. will continue to maintain preeminence in these powerful, innovative endeavors;
2. Supporting the use of mathematical results and acumen in order to increase the capabilities of the various scientific and engineering disciplines and to help advance science and technology in the United States; and
3. Providing top-quality education and skills in the mathematical sciences at all levels and for all members of the student population to ensure that the United States will continue to have a scientifically and technically trained work force and an informed citizenry.

- *Fulfilling these responsibilities is doubly challenging in the current changed environment*. The end of the Cold War has increased the national emphasis on economic competitiveness and pressing social concerns, health care issues, and environmental needs. One consequence of this shift in national priorities is that a strong scientific base, as an end in itself, is not as high a priority as it was. Another change affecting the environment for science, and for the mathematical sciences, involves the national resolve to balance the federal budget and to cut government spending.
- *There are several implications of these changes*. In the near future there may be less public money (in constant dollars) for support of universities and of scientific research. Also, the scientific community may have to justify much of its research as contributing to societal goals. For example, quality education, especially K-12

education, for all segments of society remains high on the national agenda and is a goal to which the scientific community is expected to contribute. What support—both political and financial—there now is for the mathematical sciences may well diminish if the community's responsibilities are not met. On the other hand, efforts to meet those responsibilities can provide opportunities for additional resources.

- *Action might be taken by different cohorts within the mathematical sciences community on various fronts*. For example, it is clear that much of the educated public, including the scientifically educated public, is unaware of new discoveries in the mathematical sciences and their importance. The general public also is not made aware of the fundamental contributions of the mathematical sciences to many advances in science and technology. In the general technical community, there is a sense that the mathematical sciences community could do more to accelerate the impact of mathematical sciences research on science, technology, and society in general.
- *The mathematical sciences community has not been very successful in the important area of attracting underrepresented minority groups, and the disproportionately small number of women at many levels in the mathematical sciences is self-evident*. By identifying broader career opportunities in the mathematical sciences, some of the obstacles to attracting these groups to the mathematical sciences might be overcome.
- *A further area where mathematical sciences involvement could be increased is in the education of all undergraduates*. It is important for mathematical sciences departments to strengthen their scientific and educational ties with the university community at large. It is also important that the full mathematical sciences community enhance its scientific and educational ties with the general scientific and technological community. The pervasiveness of the mathematical sciences in those diverse areas makes this both a responsibility and an opportunity.
- *In light of the present environment for science and the direction that continuing change in that environment will likely take, how the mathematical sciences community now acts could be crucial to its future strength and health, and so deserves urgent attention*. **The mathematical sciences community now faces serious challenges in a changing and demanding environment that warrant quick and effective responses. The health of the mathematical sciences community and how well it fulfills its national responsibilities depend on its ability to resolve these issues**.
- *Clearly, a successful resolution of these issues requires the active participation of mathematical sciences departments, the professional societies, and the community as a whole*

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APPENDICES

Appendix A

Workshop Participants, Agenda, and Discussion Materials

Participants

Alejandro Adem
Professor
Mathematics Department
University of Wisconsin

Louis Auslander
Professor
Mathematics Graduate School
City University of New York

M. Salah Baouendi
Professor
Mathematics Department
University of California-San Diego

Hyman Bass
Professor
Department of Mathematics
Columbia University

Lynne Billard
Professor
Department of Statistics
University of Georgia

Spencer Bloch
Professor
Department of Mathematics
University of Chicago

Joseph E. Brandenburg
Principal Engineer
Intel Corporation

William Browder
Professor
Department of Mathematics
Princeton University

Lawrence Brown
Professor
Department of Statistics
University of Pennsylvania

Robert L. Bryant
Professor
Department of Mathematics
Duke University

Jennifer Tour Chayes
Professor
Mathematics Department
University of California-Los Angeles

Ralph Cohen
Professor and Chair
Department of Mathematics
Stanford University

James Crowley
Executive Director
Society for Industrial and Applied Mathematics

Ronald G. Douglas
Executive Vice President,Provost
Texas A&M University

Robert Fefferman
Professor and Chair
Department of Mathematics
University of Chicago

Walter Feit
Professor
Department of Mathematics
Yale University

Avner Friedman
Director
Institute for Mathematics and Its Applications
University of Minnesota

Donald Goldfarb
Professor and Chair
Department of Industrial Engineering and Operations Research
Columbia University

David Goldston
Legislative Director
Office of Representative Sherwood Boehlert
U.S.House of Representatives

John Guckenheimer
Professor
Department of Mathematics
Cornell University

Philip Hanlon
Professor
Department of Mathematics
University of Michigan

Richard Herman
Dean,College of Computer,Mathematical and Physical Sciences
University of Maryland-College Park

Ettore F. Infante
Senior Vice President for Academic Affairs
University of Minnesota

Arthur Jaffe
Professor
Department of Mathematics
Harvard University

Stephen Kennedy
Assistant Professor
Mathematics Department
Carleton College

Jon R. Kettenring
Executive Director
Software Technology Integration
Bellcore

Joseph J. Kohn
Professor and Chair
Department of Mathematics
Princeton University

Donald J. Lewis
Director
Division of Mathematical Sciences
National Science Foundation

William James Lewis
Professor and Chair
Department of Mathematics and Statistics
University of Nebraska-Lincoln

Mary Montgomery Lindquist
Professor
School of Education
Columbus College

Robert MacPherson
Professor
School of Mathematics
Institute for Advanced Study

Marilyn E. Mays
Professor
Department of Mathematics
North Lake College

David W. McLaughlin
Professor and Director
Courant Institute of Mathematical Sciences
New York University

Donald St. P. Richards
Professor and Chairman
Division of Statistics
University of Virginia

Hugo Rossi
Professor
Mathematics Department
University of Utah

Michael Schrage
Science Writer
MIT Media Lab,*Los Angeles Times,*
Wired Magazine

Frank Stillinger
Technical Staff
Bell Laboratories

Dewitt Sumners
Professor
Department of Mathematics
Florida State University

John R. Tucker
Director
Board on Mathematical Sciences
National Research Council

James C. Turner, Jr
Professor and Chair
Department of Mathematics
Florida A&M University

Michael Waterman
Professor
Department of Mathematics
University of Southern California

Andrew B. White, Jr
Program Director,High Performance Computing and Communications
Advanced Computing Laboratory
Los Alamos National Laboratory

Shmuel Winograd
IBM Fellow
IBM T.J.Watson Research Center

Margaret H. Wright
Distinguished Member,Technical Staff
Bell Laboratories

Agenda

Friday, May 17, 1996

11:00 am-1:00 pm	Registration
1:00 to 1:30	**INTRODUCTORY COMMENTS**
	Avner Friedman, Chair, Board on Mathematical Sciences
1:30 to 2:15	**NEW OPPORTUNITIES**
	Expectations and New Opportunities at DMS
	Donald J. Lewis, Director,
	Division of Mathematical Sciences, National Science Foundation
	Communicating Mathematics to the Public
	Michael Schrage, MIT Media Lab/Sloan School; Columnist, Los Angeles Times
2:15 to 2:30	Discussion
2:30 to 3:15	**THE WASHINGTON ENVIRONMENT**
	The Washington Environment Viewed from OSTP
	Ernest Moniz, Associate Director for Science, Office of Science and Technology Policy
	A View from Capitol Hill
	David Goldston, Legislative Director for Representative Sherwood Boehtert
3:15 to 3:30	Discussion
4:00 to 4:45	**THE VIEW OF SCIENTISTS**
	Physical Scientists Are from Mars, Mathematicians Are from Venus; How on Earth Can We Communicate?
	Frank Stillinger, Member of Materials Chemistry Research Department Technical Staff, Bell Laboratories
	The Growing Impact of Mathematics in Molecular Biology
	Michael S. Waterman, Professor of Mathematics and Biological Sciences, University of Southern California
4:45 to 5:30	**PERSPECTIVES ON EDUCATION**
	Education (for the Public and Students), and the Mathematical Scientist's Role in It
	Hyman Bass, Adrain Professor of Mathematics, Columbia University
	Educating Mathematical Sciences Graduate Students
	Ronald Douglas, Executive Vice President and Provost, Texas A&M University

6:30 to 8:00	Dinner
	Mathematical Sciences and National Needs: Roles and Implications
	Judith S. Sunley, Assistant to the Director for Science Policy and Planning, National Science Foundation
8:00 to 10:00 pm	Initial Meeting of Break-out Groups to Discuss General Theme: Needed Changes and Action Guidelines to Achieve Them
Saturday, May 18	
9:00 to 10:00 am	Reports from Friday Evening Small Group Discussions
10:00 to 12:00 noon	Break-out Session on Specific Topics
12:00 to 1:30 pm	Lunch
	A View of Major Trends at Research Universities
	Ettore Infante, Senior Vice President for Academic Affairs, University of Minnesota
1:30 to 4:00	Break-out Discussions, and Preparation of Written Reports Guidelines That Will Work and Be Acted On
4:30 to 5:30 pm	Written Reports from Small Groups and Discussion
Sunday, May 19	
9:00 to 10:00 am	Presentation of Draft Summary Document
	Group Leaders, Avner Friedman, and John Tucker
10:00 to 11:45	Reaction to the Draft, Discussion, and Modifications/Adjustments to Gain Consensus
11:45 to 12:00	Closing Comments
	Avner Friedman
12:00 noon	Adjourn

DISCUSSION MATERIALS

Small Group Discussion[1]

Each of the four groups will focus on one of these topics (recognizing that the topics have areas of overlap, and that all have as a subtheme undergraduate education):

Discussions should address

Needed Changes & Action Guidelines to Achieve Them

How to Assure that Guidelines Will Work and Be Acted On

1. Communication (within and outside the discipline)

Group Leader: Margaret H. Wright (Bell Labs)

The mathematical sciences occupy an unusual position in the public consciousness. Essentially all adults studied mathematics in school, but many disliked it; most people would say that mathematics is important, but, if pressed, they might have difficulty explaining why; abstraction is one of the chief virtues of the mathematical sciences, but nonexperts understand science best if it is described using concrete examples related to their daily lives. Our community therefore has an especially challenging obligation to clarify the many ways in which the mathematical sciences are beneficial to society. We have an equally compelling responsibility to convey to our colleagues in other disciplines the contributions that research and education in the mathematical sciences have made and continue to make to advancing their fields. Our community needs to articulate its contributions in three areas: general education (K-12); undergraduate and graduate education; and research.

Possible questions to be addressed include:

- What efforts are being made in public awareness?
- How effective have these been? Which have been most and least successful? What lessons can be learned from these experiences?
- Which public awareness activities can be expected to appeal to various segments of the public?
- What kinds of activities in public awareness are mathematical scientists willing to support and join? How can such activities be organized for maximum impact and participation?
- How can we in the mathematical sciences build stronger connections with other disciplines and with nonacademic organizations?
- How can our community work effectively with other disciplines to convey the benefits of education and research in the mathematical sciences?

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[1] These guidelines (as well as the background materials mentioned at the beginning of the summary on page 51 of the main text) were distributed to all invited speakers and participants two weeks before the workshop.

2. How to Improve the Educational Program

Group Leader: Hugo Rossi (University of Utah)

Last year, the National Academy Press published a report of the Committee on Science, Engineering, and Public Policy (COSEPUP) entitled *Reshaping the Graduate Education of Scientists and Engineers* (NAS/NAE/IOM, 1995). It is appropriate to take this report as background for the discussion.

Findings of the COSEPUP report:

1. There is a changing pattern of employment of graduate students:

—growth in faculty positions is slowing down; expect a reduction in demand for traditional researchers;
—new R&D needs in industry result in emerging production, service, and information enterprises;
—in government labs, research foci are shifting (i.e., from defense to energy), and the labs are challenged to build linkages with industry and universities.

2. There are far more seekers of jobs as professors and primary researchers than there are positions; however, the number of positions in applied R&D is increasing.

—about 50% of new Ph.D.s are employed in academic departments; the number of Ph.D.s in mathematics fell during the 1970s, but since has increased to boom-time levels. Almost 50% of Ph.D.s awarded in the 1990s have gone to foreign students. Employment of Ph.D.s in academic positions has steadily decreased over the past 12 years, while other employment has remained level.

3. We have not, as a nation, paid adequate attention to the function of the graduate schools in meeting the country's needs. The simplifying assumption has been that the primary mission of graduate programs is to produce the next generation of researchers.

Recommendations of COSEPUP:

1. Offer a broader range of academic options that allow students to gain a wider variety of skills.

—mechanisms of support should include education/training grants to departments.

2. Provide better information and guidance, directed toward possible career goals.
3. Devise a National Human Resource Policy, beginning with a national discussion of goals and policy, system characterization, and contemporary issues.
4. Curricular:

—"tighten" time to degree. At present, the average time to degree in the mathematical sciences is about 6 years.
—provide options: at the time of qualifying examinations, students should choose from (a) a viable master's degree, (b) proceed to a Ph.D. and a position in research, (c) a "designed" dissertation for work in nontraditional fields.

—create interdisciplinary programs.
—introduce internships outside academics.
—devise and implement steps to increase participation of women and minority groups.

Questions and Possible Action Items

1. Accept, reject, or modify with some detail the recommendations of COSEPUP for the mathematical sciences.
2. How can the mathematical sciences community develop mechanisms for information about successful programs, and develop networks of people who will help provide advice?
3. Is there a changing pattern of how mathematics research is done: individual to group and interdisciplinary? How can graduate programs encourage versatility?
4. What curriculum changes are high priorities?

—time to degree
—interdisciplinary programs
—computer competency
—teaching competency

5. Should there be a strengthened and revitalized professional master's degree?
6. Should there be optional internships (in other departments or in industry)? What should be their character, and how should their implementation be facilitated?
7. Should NSF broaden its sabbatical programs (for faculty or students); should the mathematical sciences move toward a postdoctoral system, as exists in other sciences?

3. Shrinking Funding (external and internal), and What Should Be Done in Light of It

Group Leader: Arthur Jaffe Harvard University)

The NSF Division of Mathematical Sciences is under stress because funding (in recent years, in constant dollars) is diminishing. Mission-oriented research funding agencies are even more focused on deliverables. Advantage needs to be taken of interdisciplinary avenues and information provided on what possibilities are coming, what has worked, and what people are doing; guidance is needed on how funds should be spent.

Possible questions to be addressed:

- Where are the new opportunities for mathematical sciences research (e.g., mathematics with materials science, statistical analysis and modeling of DNA, topological and geometric methods for molecular biology, software engineering, high-performance computing, and combining information—including linkage of databases, combining

results from independent studies, and in geographic information systems for spatial analysis)?
- How can the mathematical sciences community encourage entrepreneurs?
- How can the mathematical sciences community set in place mechanisms for lecture series, reports, symposia, and committees to explore ways to further increase research opportunities?
- How can the community be made aware of promising opportunities?
- How can more of the community be positioned to take advantage of those opportunities?
- What are appropriate modes of support?

4. Evaluating Performance (both educational and interdisciplinary)

Group Leader: Robert MacPherson (Institute for Advanced Study)

Traditional evaluation in most departments is based first on research, then on education and on interdisciplinary work. Since such a criterion does not encourage faculty who wish to contribute more in the way of teaching innovations, interfacing with other departments, and engaging in extra-university activities, questions naturally arise. While change is needed with regard to what is rewarded and what the culture values (JPBM, 1994), it has to come from the grassroots.

Possible questions to be addressed:

- How to develop a more appropriate evaluation process?
- What are the impediments to effecting such a process?
- How can departments overcome such impediments?
- What role should NSF play here?
- What ways of evaluating teaching, or interdisciplinary research are working?
- How can the value of interdisciplinary work be better conveyed?
- What are some specific actions that can be taken concerning this?

References

Joint Policy Board for Mathematics (JPBM). 1994. *Recognition and Rewards in the Mathematical Sciences*. Committee on Professional Recognition and Rewards. Providence, R.I.: American Mathematical Society.

National Academy of Sciences/National Academy of Engineering/Institute of Medicine (NAS/NAE/IOM). 1995. *Reshaping the Graduate Education of Scientists and Engineers*. Committee on Science, Engineering, and Public Policy. Washington, D.C.: National Academy Press.

Appendix B

The "Douglas" Report's Executive Summary

Although the United States is considered a world leader in mathematical sciences research and in doctoral and postdoctoral education, concern is growing about whether the needs of the profession and of an increasingly technological society are being met. Many doctoral students are not prepared to meet undergraduate teaching needs, establish productive research careers, or apply what they have learned in business and industry. This inadequate preparation, continuing high attrition, and the declining interest of domestic students, the inadequate interest of women students, and the near-absent interest of students from underrepresented minorities in doctoral study are problems that transcend the current difficult job market.

The charge to the Committee on Doctoral and Postdoctoral Study in the United States was to determine what makes certain programs successful in producing large numbers of domestic Ph.D.s, including women and underrepresented minorities, with sufficient professional experience and versatility to meet the research, teaching, and industrial needs of our technology-based society. The committee based its findings on site visits to a diverse set of programs in 10 universities carried out in late 1990 and early 1991. These programs were in both small and large, and in both public and private, universities. They were also geographically diverse. They were all in the "top 100" and included four departments in the "top 20."

The audience to which the report speaks is all U.S. doctoral and postdoctoral programs in the mathematical sciences, and, in particular, those programs that have limited human and financial resources. The report suggests that even with limited resources success can be achieved if, among other things, a program focuses its energies rather than trying to implement a "standard" or traditional program that covers too many areas of the mathematical sciences. It also notes that departments with the best faculty do not necessarily have the most successful doctoral and postdoctoral programs. A quality faculty is necessary for a good program, but of equal importance are students and researchers that can benefit from the program.

In this report, a "successful" program is understood to be one that accomplishes the following two objectives.

- All students, including the majority who will spend their careers in teaching, government laboratories, business, and industry rather than in academic research, should be well prepared by their doctoral and postdoctoral experience for their careers.
- Larger percentages of domestic students, and, in particular, women and underrepresented minorities, should be attracted to the study of and careers in the mathematical sciences.

Note: Reprinted from *Educating Mathematical Scientists: Doctoral Study and the Postdoctoral Experience in the United States* (National Academy Press, Washington, D.C., 1992).

In its site visits, the Committee on Doctoral and Postdoctoral Study in the United States looked for features that were present in successful programs as well as for elements that were detrimental to quality education. The committee noted that successful programs possessed, in addition to the *sine qua non* of a quality faculty, the following three characteristics:

1. A focused, realistic mission
2. A positive learning environment
3. Relevant professional development

A positive learning environment is an environment that provides the assistance, encouragement, nurturing, and feedback necessary to attract and retain students and to give them an education appropriate for their future careers.

The findings of the committee are as follows:

- There are several different models (missions) for programs, including

- the standard model, which supports research in a broad range of areas, offers depth in each one, and has as its goal preparation for careers at research universities, and
- specialized models, such as the subdisciplinary model, the interdisciplinary model, the problem-based model, and the college-teachers model, which were seen to alleviate two large human resource problems, recruitment and placement, and to be conducive to clustering of faculty, postdoctoral associates, and students, a practice that helps create a positive learning environment and promote relevant professional development.

- Both standard and specialized programs can be successful. However, programs that do not have the human or financial resources to run a successful standard program should consider whether a specialized model might better fit their needs.
- New Ph.D.s with a broad academic background and communication skills appropriate for their future careers are better able to find jobs.
- Active recruiting increases the pool of quality students. It does not just reapportion the pool. It also increases the number of women and underrepresented minorities. Students with strong mathematical backgrounds have a choice of studying mathematical sciences, physical sciences, engineering, law, medicine, and other areas. More of them can be attracted to the mathematical sciences.
- Clustering faculty, postdoctoral associates, and doctoral students together in research areas is a major factor in creating a positive learning environment.
- A positive learning environment is important to all doctoral students but is crucial for women and underrepresented minorities.

- All departments, including those characterized as elite and selective, need to provide a supportive learning environment.
- Doctoral students and postdoctoral fellows should receive broad academic preparation appropriate for their future careers in research universities, teaching universities, government laboratories, business, and industry.
- Doctoral students and postdoctoral fellows should learn teaching skills and other communication skills appropriate for their future careers.
- The number of postdoctoral fellowships in the mathematical sciences should be greatly increased so that such positions can be viewed as the logical next step after completion of the doctorate for the good student, not as a highly competitive prize for a select few. More postdoctoral fellowships should have applied, interdisciplinary, or pedagogical components.

Changing the American doctoral and postdoctoral system in the mathematical sciences so that it responds better to the needs of the profession, students, and the society is a task that requires the cooperative efforts of faculty, departments, professional societies, and federal agencies. The departments at research universities have a special responsibility to raise the level and increase the knowledge of talented but underprepared entering American doctoral students. Federal agencies should continue their programs and also increase their awareness of the impact of their programs on the doctoral and postdoctoral system. Professional societies should be involved in monitoring change in the universities, the agencies, and the community. But action, if it starts at all, will start from the faculty. The faculty should be aware that creating and maintaining a successful doctoral/postdoctoral program will require additional effort and time. The long-term benefits to the department, the students, and the society are clearly worth the effort.

Appendix C

National Science Foundation Workshop Summary Report

Preface

On June 5 and 6, 1995, the Directorate of Mathematical and Physical Sciences (MPS) of the National Science Foundation (NSF) sponsored a workshop on graduate education in the mathematical and physical sciences. Its purpose was to bring together leaders from MPS disciplines to examine the current practices used to prepare young people for careers in science, and to suggest strategies appropriate for the next decade. I am pleased to share with you the findings and recommendations of the workshop.

In many ways, the environment in which today's graduate students and postdoctorals will work is going to be very different from what we knew as young scientists. Federal investments in research will remain constant or decline, at least over the near term. The role of research in industry is changing, especially the contributions expected of the central laboratories of large companies. Added to this must be the realization that the long-sought goal of greater participation by underrepresented groups is being achieved more slowly than we might hope.

In response, we must recognize this new reality and optimize the use of limited human and capital resources. Young scientists should not only be trained to advance intellectual frontiers, but also to meet the changing needs of the universities, companies, and research organizations that will employ them. They should recognize opportunities in their own fields and connections to other disciplines. They should be able to describe the beauty and importance of their work as easily to a group of high school students as to their peers.

John Armstrong, the former director of research at IBM who has given voice to many of these concerns previously, provided skillful and creative leadership both during the meeting and as chair of the steering committee which oversaw its planning. The committee was ably supported by a working group of MPS staff. The workshop also benefited from strong intellectual support by NSF Director Neal Lane and Deputy Director Anne Petersen.

Discussion at the sessions was lively and thoughtful. It was not a "typical" NSF workshop in the sense of being focused on a specific area of scientific research. Yet it followed a model that has been used many times and in which NSF believes strongly: identifying an important issue being discussed in the scientific community and creating a forum that furthers the dialogue.

The workshop serves as part of a broader strategic planning effort within MPS. While the opinions expressed in this report are those of the speakers and invited participants and do not

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Note: Summary from *Graduate Education and Postdoctoral Training in the Mathematical and Physical Sciences*, National Science Foundation, Arlington, VA, 1996, publication NSF 96-30. Reprinted from the World Wide Web at http://www.nsf.gov:80mps/workshop.htm.

represent NSF policy, I consider them an important step for developing an agenda that will help ensure the health of the mathematical and physical sciences and provide greater opportunities for the next generation of scientists. These recommendations are currently under study within NSF. I invite you to participate in this ongoing dialogue.

William C. Harris
Assistant Director Directorate for
Mathematical and Physical Sciences

Introduction

The heightened international economic and technological competitiveness of the post-Cold War world and the growing influence of domestic fiscal stringency are forcing the research university community to adjust to a changing environment. In particular, these new forces affect the training of graduate students in the physical sciences and mathematics, and so the Directorate of Mathematical and Physical Sciences (MPS) of the National Science Foundation (NSF) convened a Workshop to examine these forces and to consider issues that impact how MPS carries out its responsibilities as an important supporter of graduate education and research.

Although the infrastructure for training Ph.D.'s in the mathematical and physical sciences in the United States has been extremely successful, this infrastructure has only now begun to respond to the demands of the changed environment. The MPS-sponsored workshop included representatives of academia (faculty, administrators, and students), industry, professional societies, national laboratories, government agencies, and other stakeholder institutions to examine current approaches to graduate and postdoctoral training. This document is the Summary Report of that Workshop.

As is the case for all MPS-sponsored workshops, this report is meant to assist the Directorate in its planning and interactions with the scientific community. It was understood by the participants that any changes in MPS programs and procedures will have to be discussed by the MPS Advisory Committee and possibly approved by NSF management and the National Science Board before going into effect. It was moreover the consensus of the participants that any eventual changes be gradual and be preceded by a period of experimentation in which groups in the MPS community are invited to propose innovations and changes in the conduct of graduate training.

After discussions in separate groups, the participants met in plenary session and endorsed the following recommendations.

1. Mechanisms should be found to encourage a broadening of the training and educational experience of MPS graduate students.
2. Mechanisms should be examined for shortening the average time to the Ph.D. degree in the MPS fields.

3. Increased use should be made of periods of off-campus experience, such as industrial internships.
4. Efforts should be made to decrease gradually the proportion of graduate students funded as Research Assistants and to increase gradually the proportion funded by other mechanisms, including traineeships and fellowships, as well as novel, collective modes of support.

We now discuss the findings and recommendations of the Workshop in more detail.

Findings

In the discussions at the MPS Workshop, participants identified a number of concerns about graduate training as it currently exists:

1. The public resources available to support these activities are likely to decrease in the immediate future.
2. The skills and knowledge acquired by new Ph.D.'s are too narrowly focused, and are not adequately applicable to the diverse business and industry environments in which most Ph.D. scientists actually work.
3. Over the past several decades, an increasing emphasis has been placed on the research component of the graduate experience, sometimes at the expense of the best interests of the student.
4. Diversity among students in graduate school has not been achieved to a satisfactory degree. Specifically, women, minorities and other underrepresented groups have yet to achieve parity within this population.

These findings are discussed in detail below.

(1) Resources

An overarching concern for all involved in graduate education and postdoctoral training in this country is the decline in Federal R&D budgets.

However, the participants strongly affirmed the importance of continued government support of these endeavors, even at a somewhat reduced level. Research done as part of graduate training directly affects areas important to all Americans and is justifiably supported by taxpayers. These areas include the environment, health care, safety, national security, and the technical infrastructure that is crucial for the innovation and increases in productivity that are the ultimate sources of a rising standard of living.

Thus, given both the crucial importance of graduate research and of education and the impending fiscal austerity, the workshop affirmed that it was a matter of the highest importance for the NSF/MPS to find ways to increase the return to the Nation on its investment in our fields.

(2) Breadth of Skills and Knowledge

There is no doubt that students in MPS disciplines who obtain Ph.D.'s from the research-oriented universities in this country are among the best prepared and most successful scientists in the world. The emphasis over the past half-century on physical sciences research has borne fruit, not only in spectacular advances in science, but also in the form of a myriad of new technologies and industries and in a cadre of highly trained individuals with the analytical and problem-solving skills needed to perform cutting edge work in many fields.

Often, however, these students are unaware not only of the options available to them outside of academia, but also of the applicability of the skills they have acquired through their graduate education to fields other than the one they have "trained" to enter. Moreover, students are finding that the jobs they have trained for are not as abundant as in the past in a number of the MPS disciplines.

On one hand, students are becoming increasingly specialized and compartmentalized in their educational and research pursuits. On the other hand, they are not usually encouraged to acquire the particular skills (interpersonal communication, management, or business-oriented skills) that would help them succeed in careers outside their field of preparation.

(3) The Balance between Education and Research

Many of these trends have been exacerbated by the funding process used at NSF and in other Federal agencies. Since the main criterion for judging grant applications has traditionally been the quality of the research to be performed, along with the success of past research, this is necessarily where the attention of grant applicants must be focused. Not only does this affect the principal investigators, who may believe they are expected to give lower priority to other aspects of the education of their students in order to keep the funding pipeline open, but it affects graduate and postdoctoral students themselves, who perform most of the labor involved in such research and who are often effectively discouraged from spending time on other educational pursuits not directly involved in their advisor's research project.

The current funding mechanism (where graduate students are supported primarily by Research Assistantships) also has the effect of allowing a lengthening of the time to obtain a Ph.D. Successful researchers are understandably unwilling to lose graduate students when they have finally become highly productive, and these students may in turn, prefer the protected, known world of the university over a usually unknown "outside" world. In neither case does the proposal and grant process take time to Ph.D. into account.

The Workshop participants also noted that a "Ph.D. or nothing" atmosphere has developed, so that the doctoral degree is often considered a minimum requirement for meaningful employment in the MPS fields. In addition, the perception of the master's degree in MPS fields as a second-rate achievement probably deters many students from entering these fields.

(4) Diversity of the Graduate Student Population

The limited involvement of women, minorities, and other underrepresented groups in mathematical and physical sciences is a long-standing phenomenon. While the Workshop participants recognized that much progress has been made to include these groups in the graduate experience, much work remains to be done. Specifically, it was noted that colleges and universities are increasingly able to attract these students into the "pipeline," but are less successful in shepherding them into and through the graduate and postdoctoral stages.

The mix of foreign and U.S. students going to graduate school was also a concern. While the scientific achievements and contributions to the country of both groups are strong, there has been a noticeable decline in the fraction of U.S. students participating in the graduate experience. This trend, coupled with the dwindling availability of resources for the support of all graduate education, has disturbing implications for the long-term health of the U.S. graduate programs in the MPS disciplines.

Recommendations

The participants at the Workshop were divided into four groups to discuss issues in particular areas of concern with respect to the graduate experience: career issues, support mode issues, educational issues, and demographic issues. Although these groups were charged with examining supposedly different aspects of the problem, a final presentation of the recommendations from each group showed a high degree of uniformity about what are the key issues and widespread consensus on recommendations.

Each general recommendation is listed below, followed by a brief supporting discussion.

(1) There should be a move to broaden the intellectual content and increase the diversity of skills acquired during Ph.D. training.

The Workshop participants noted that a more diverse mix of skills and abilities would better enable new Ph.D.'s to take advantage of the changing career market. This diversity could be fostered through encouraging the reinstatement or reinvigoration of breadth requirements, such as "minors." In these programs, schools could offer courses designed to foster interdisciplinary training or facilitate experiences that provide preparation for specific, nonacademic technical careers.

Another way to adapt graduate training to fit current requirements would be the development of professional-level master's programs in MPS disciplines, which Workshop participants likened to professional engineering degrees or the MBA. Such programs could well be linked to specific industrial or commercial career paths and would thus require significant participation from stakeholders in these sectors.

Finally, in an effort to increase the ability of MPS graduate students to choose a broader range of careers, the participants stressed that the education of these students should involve more attention to the development of "soft skills," such as ethics, business, and financial skills, and most of all communications skills, such as writing, presenting, and listening.

(2) Mechanisms for shortening the average time to Ph.D. should be examined.

The Workshop participants noted that the average time required for the completion of doctoral study has increased during recent decades. This was also a finding of the recent COSEPUP report on graduate education. Although this observation at the Workshop was based mostly on anecdotal evidence and the time-to-degree constraint would naturally differ from field to field (and even from school to school), this was nonetheless identified as a general trend.

The concern over the time required for completion of degrees centered both on the welfare of the student and on the additional cost to the Nation of long, publicly supported stays in graduate school. While these long stays may not pose a problem to those students pursuing positions in academia, students attempting to enter commercial or other sectors often find themselves at a disadvantage to those without Ph.D.'s but with a number of years of practical, on-the-job experience.

The Workshop participants did not agree to recommend that time constraints be placed on the completion of doctoral degrees across the board because longer periods of study and research appear to be necessary in some fields. Rather, they suggested that NSF examine "best practice" data from programs around the country and encourage the shortening of time-to-degree periods where feasible.

(3) Increased use of off-campus internships and other real-world experiences.

In the interest of producing more broadly educated students with more wide-ranging career expectations and capabilities, the participants recommended the incorporation of an option for some sort of internship, or real-world work experience, into the traditional education of graduate students.

Particularly relevant to this recommendation is the success of the NSF "GOALI" program (Grant Opportunities for Academic Liaison with Industry), as well as government laboratory exchange programs, which have enabled students to gain firsthand knowledge of the culture, environment, and intellectual challenges present beyond academia. These programs have been particularly useful within the framework of the undergraduate engineering experience, but the participants believe they are applicable to the broader spectrum of MPS disciplines and to Ph.D.-level training.

To facilitate the development of these programs, the participants recognized that a significant commitment on the part of both industry officials and the university research community must be obtained. To this end, NSF should be encouraged to experiment with awards to grant applicants who propose such arrangements in a realistic and effective fashion.

(4) Gradual shift in graduate student support mechanisms.

Currently, the bulk of graduate student support provided by the Foundation is in the form of awards to individual investigators, who use these funds in part to support graduate students. Many participants agreed that this often has had the unintended consequence of limiting the areas in which students take courses and acquire experience.

The Workshop recommended that MPS experiment with means to increase gradually the fraction of graduate students supported on fellowships and traineeships. Further, NSF should encourage members of the MPS community in academia to propose new institutional, "thematic" funding mechanisms for graduate student training and support that would involve collective responsibility for groups of students.

Funds could be awarded to entire departments, to combinations of departments, or to theme-oriented entities that would allocate resources to students themselves. This would have the effect of allowing departments, or other groups, to take greater ownership of the overall quality of graduate education. The criteria for making awards would have to guarantee that special, new efforts would be made to achieve the desired educational improvements. In addition, NSF could reward and encourage such "collective proposals" that exhibit success in the recruitment and retention of students from underrepresented groups. including Women, minorities, and, where applicable, domestic students.

Conclusion

Above all, the Workshop offers these recommendations with the intention of stimulating further debate in the MPS community and experimentation by the NSF on ways and means to improve graduate education. These suggestions are not intended to be the final word on any particular issue, and the participants stressed that they did not endorse a "one size fits all" analysis or treatment. Improvements to graduate education can be structured in many ways. To be effective, most changes must come from within the universities and academic departments themselves. However, it was also the conclusion of the Workshop that NSF should play a constructive role in sponsoring experiment and change in graduate education.

As the world of science and engineering is changing, so, too, must the Foundation adapt to these changes. The workshop participants hope that their findings and recommendations will be helpful to the NSF community as a whole.